KEY

GSE Algebra II
Inverse Functions and Function Composition REVIEW

Name________________________
Date________________ Period_____

## Part I – Functions and Relations

✓1. A function is a special type of relationship where each input(x) has only one output(y).
All functions will pass the vertical line test.

✓2. A function is called "one-to-one" **if and only if** each element in the domain maps to a unique element in the range. All one-to-one functions will pass the horizontal line test.

(NOTE: For a function to be called one-to-one, the inverse must be a function.)

For questions 3-6, determine if the relation is a function and if it is one-to-one. Write "YES" or "NO".

✓ 3) (1,2), (2,3), (5,6)

Yes, it is a function. (No x-coordinates repeat)

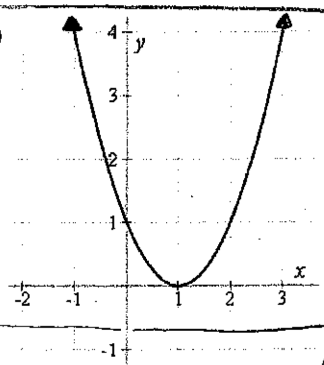
Yes, it is one-to-one. (No y-coordinates repeat)

✓ 4) (1,1), (2,1), (3,1)

Yes, it is a function. (No x-coordinates repeat)

No, it is not one-to-one. (y-coordinates repeat)

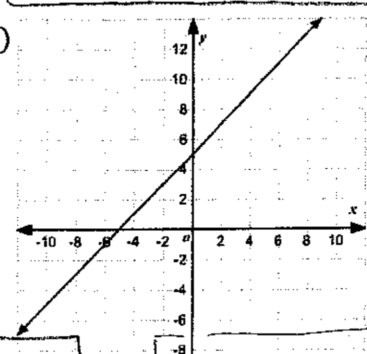
✓ 5)



Yes, it is a function. (passes vertical line test)

No, it is not one-to-one. (does not pass horizontal line test)

✓ 6)



Yes, it is a function. (passes vertical line test)

Yes, it is one-to-one. (passes horizontal line test)

## Part II - Inverses

For questions 7-14, find the inverse of the given function. For the graphs, graph the inverse.

| | |
|---|---|
| ✓7) $y = 2x - 7$<br>$x = 2y - \cancel{7}$<br>$+7 \quad \cancel{+7}$<br>$\frac{x+7}{2} = \frac{\cancel{2}y}{\cancel{2}}$<br>$\boxed{y = \frac{x+7}{2}}$<br>OR<br>$\boxed{y = \frac{1}{2}x + \frac{7}{2}}$ | ✓8) $f(x) = \frac{x-4}{3}$<br>$3x = \frac{y-4}{\cancel{3}} \cdot \cancel{3}$<br>$3x = y - \cancel{4}$<br>$+4 \quad \cancel{+4}$<br>$\boxed{y = 3x + 4}$ |
| ✓9) $y = x^2 + 1$<br>$x = y^2 + \cancel{1}$<br>$-1 \quad \cancel{-1}$<br>$\pm\sqrt{x-1} = \sqrt{y^2}$<br>$\boxed{y = \pm\sqrt{x-1}}$ | ✓10) $g(x) = \sqrt[3]{2x-5} + 9$<br>$x = \sqrt[3]{2y-5} + \cancel{9}$<br>$-9 \quad \cancel{-9}$<br>$(x-9)^3 = (\sqrt[3]{2y-5})^3$<br>$(x-9)^3 = 2y - \cancel{5}$<br>$+5 \quad \cancel{+5}$<br>$\frac{(x-9)^3+5}{2} = \frac{\cancel{2}y}{\cancel{2}}$<br>$\boxed{y = \frac{(x-9)^3+5}{2}}$ |

KEY

11) $y = \dfrac{7}{2+x}$

$(2+y)x = \dfrac{7}{\cancel{2+y}} \cdot \cancel{(2+y)}$

$\dfrac{(2+y)\cancel{x}}{\cancel{x}} = \dfrac{7}{x}$

$\cancel{2}+y = \dfrac{7}{x}$
$-\cancel{2} \qquad -2$

$\boxed{y = \dfrac{7}{x} - 2}$

12) $f(x) = 3x^3$

$\dfrac{x}{3} = \dfrac{\cancel{3}y^3}{\cancel{3}}$

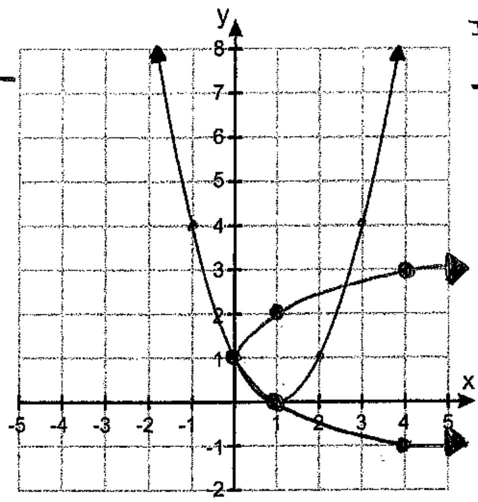
$\sqrt[3]{\dfrac{x}{3}} = \sqrt[3]{y^3}$

$y = \dfrac{\sqrt[3]{x}}{\sqrt[3]{3^1}} \cdot \dfrac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}}$

$\boxed{y = \dfrac{\sqrt[3]{9x}}{3}}$

13)

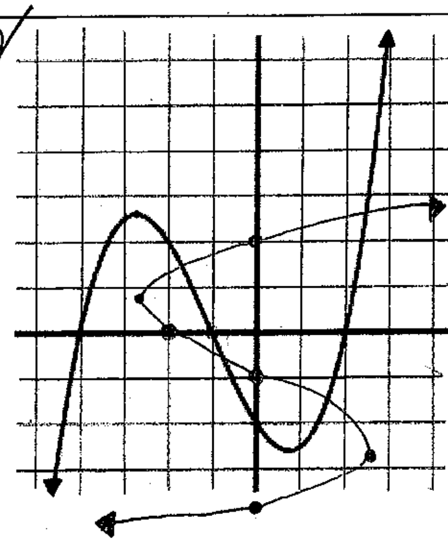
| x | y |
|---|---|
| -1 | 4 |
| 0 | 1 |
| 1 | 0 |
| 2 | 1 |
| 3 | 4 |




INVERSE

| x | y |
|---|---|
| 4 | -1 |
| 1 | 0 |
| 0 | 1 |
| 1 | 2 |
| 4 | 3 |

14)



| x | y |
|---|---|
| -4 | 0 |
| -2.8 | 2.6 |
| -1 | 0 |
| 0 | -2 |
| 0.8 | -2.6 |
| 2 | 0 |

INVERSE

| x | y |
|---|---|
| 0 | -4 |
| 2.6 | -2.8 |
| 0 | -1 |
| -2 | 0 |
| -2.6 | 0.8 |
| 0 | 2 |

For Questions 15-18, determine if the functions are inverses using composition. NOTE: PROVE $f(g(x)) = x$ AND $g(f(x)) = x$

15) $f(n) = \dfrac{-16+n}{4}$

$g(n) = 4n + 16$

$f(g(n)) = \dfrac{\cancel{-16}+4n\cancel{+16}}{4}$

$= \dfrac{\cancel{4}n}{\cancel{4}}$

$= n$ ✓

$g(f(n)) = \cancel{4}\left(\dfrac{-16+n}{\cancel{4}}\right) + 16$

$= \cancel{-16} + n \cancel{+16}$

$= n$ ✓ $\boxed{\text{Yes, inverses.}}$

16) $f(x) = 2x + 6$

$g(x) = \dfrac{1}{2}x - 3$

$f(g(x)) = 2\left(\frac{1}{2}x - 3\right) + 6$

$= x \cancel{-6} \cancel{+6}$

$= x$ ✓

$g(f(x)) = \frac{1}{2}(2x+6) - 3$

$= x \cancel{+3} \cancel{-3}$

$= x$ ✓ $\boxed{\text{Yes, inverses.}}$

17) $f(x) = 9 - \dfrac{1}{3}x$

$g(x) = 3x + 3$

$f(g(x)) = 9 - \frac{1}{3}(3x+3)$

$= 9 - x - 1$

$= -x + 8$ $\boxed{\text{No, not inverses.}}$

18) $h(x) = \sqrt[3]{2x+5} - 1$

$r(x) = (2x-1)^3 - 5$

$h(r(x)) = \sqrt[3]{2((2x-1)^3 - 5) + 5} - 1$

$= \sqrt[3]{2(8x^3 - 12x^2 + 6x - 6) + 5} - 1$

$= \sqrt[3]{16x^3 - 24x^2 + 12x - 12 + 5} - 1$

$= \sqrt[3]{16x^3 - 24x^2 + 12x - 7} - 1$

$\boxed{\text{No, not inverses.}}$

For questions 19-22, complete the compositions given:

$f(x) = 7x - 3 \qquad g(x) = x^2 + 5x - 1 \qquad h(x) = \dfrac{1}{3}x + 10$

19) $f(h(9)) =$ $h(9) = \frac{1}{3}(9) + 10$

$= 3 + 10$

$= 13$

$f(13) = 7(13) - 3$

$= 91 - 3$

$= \boxed{88}$

20) $(g \circ f)(2)$ $f(2) = 7(2) - 3$

$= 14 - 3$

$= 11$

$g(11) = (11)^2 + 5(11) - 1$

$= 121 + 55 - 1$

$= \boxed{175}$

21) $f(g(x))$ $7(x^2 + 5x - 1) - 3$

$7x^2 + 35x - 7 - 3$

$\boxed{7x^2 + 35x - 10}$

22) $(h \circ g)(x)$ $\frac{1}{3}(x^2 + 5x - 1) + 10$

$\frac{1}{3}x^2 + \frac{5}{3}x - \frac{1}{3} + 10$

$\boxed{\frac{1}{3}x^2 + \frac{5}{3}x + \frac{29}{3}}$

**ALSO, study the "Applications of Compositions" packet.**