

Factor each polynomial function, using the given information.

11.  $f(x) = x^3 - 8x^2 + 5x + 14$ ;  $f(2) = 0$

$$\begin{array}{r|rrrrr} 2 & 1 & -8 & 5 & 14 & \\ & & 2 & -12 & -14 & \\ \hline & 1 & -6 & -7 & 0 & \end{array}$$

$$x^2 - 6x - 7 \quad x=2$$

$$(x-7)(x+1)(x-2)$$

12.  $f(x) = 12x^3 + 8x^2 - 13x + 3$ ;  $x = \frac{1}{2}$  is one zero

$$\begin{array}{r|rrrrr} \frac{1}{2} & 12 & 8 & -13 & 3 & \\ & & 6 & 7 & -\frac{3}{2} & \\ \hline & 12 & 14 & -6 & 0 & \end{array}$$

$$12x^2 + 14x - 6 \quad x = \frac{1}{2}$$

$$2(6x^2 + 7x - 3)(x - \frac{1}{2})$$

13.  $f(x) = x^3 + 3x^2 - 34x + 48$ ;  $(x-3)$  is a factor

$$\begin{array}{r|rrrrr} 3 & 1 & 3 & -34 & 48 & \\ & & 3 & 18 & -48 & \\ \hline & 1 & 6 & -16 & 0 & \end{array}$$

$$(x^2 + 6x - 16)(x-3)$$

$$(x+8)(x-2)(x-3)$$

14.  $f(x) = x^4 + 6x^3 - 4x^2 - 54x - 45$ ;  $f(-5) = 0$

$$\begin{array}{r|rrrrrr} -5 & 1 & 6 & -4 & -54 & -45 & \\ & & -5 & -5 & 45 & 45 & \\ \hline & 1 & 1 & -9 & -9 & 0 & \end{array}$$

$$(x^3 + x^2 - 9x - 9)$$

$$x^2(x+1) - 9(x+1)$$

$$(x+1)(x^2 - 9)$$

$$(x+1)(x+3)(x-3)(x+5)$$

15.  $f(x) = 2x^4 - 9x^3 + 4x^2 + 21x - 18$ ;  $x = 2$  and  $x = 3$  are roots

$$\begin{array}{r|rrrrrr} 2 & 2 & -9 & 4 & 21 & -18 & \\ & & 4 & -10 & -12 & 12 & \\ \hline & 2 & -5 & -6 & 9 & 0 & \end{array}$$

$$\begin{array}{r|rrrrr} 3 & 2 & -9 & 4 & 21 & -18 & \\ & & 6 & 3 & -9 & 0 & \\ \hline & 2 & -3 & 1 & 0 & & \end{array}$$

$$2x^2 + x - 3$$

$$(2x+3)(x-1)(x-2)(x-3)$$

Find the zeros of each function, using the given information.

16.  $f(x) = x^3 + 2x^2 - 20x + 24$ ;  $(x+6)$  is a factor

$$\begin{array}{r|rrrrr} -6 & 1 & 2 & -20 & 24 & \\ & & -6 & 24 & -24 & \\ \hline & 1 & -4 & 4 & 0 & \end{array}$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)(x-2) = 0$$

$$x = 2, -6$$

17.  $f(x) = 2x^3 + 3x^2 - 3x - 2$ ;  $f(-2) = 0$

$$\begin{array}{r|rrrrr} -2 & 2 & 3 & -3 & -2 & \\ & & -4 & 2 & 2 & \\ \hline & 2 & -1 & -1 & 0 & \end{array}$$

$$2x^2 - x - 1 = 0$$

$$(2x+1)(x-1) = 0$$

$$x = -\frac{1}{2}, 1, -2$$

18.  $f(x) = 2x^3 + 11x^2 + 9x + 2$ ;  $(2x+1)$  is a factor

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 2 & 11 & 9 & 2 & \\ & & -1 & -5 & -2 & \\ \hline & 2 & 10 & 4 & 0 & \end{array}$$

$$2x^2 + 10x + 4 = 0$$

$$x^2 + 5x + 2 = 0$$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

$$x = -\frac{1}{2}, \frac{-5 \pm \sqrt{17}}{2}$$

19.  $f(x) = x^4 + 2x^3 - 14x^2 - 32x - 32$ ;  $\pm 4$  are zeros

$$\begin{array}{r|rrrrrr} 4 & 1 & 2 & -14 & -32 & -32 & \\ & & 4 & 24 & 40 & 32 & \\ \hline & 1 & 6 & 10 & 8 & 0 & \end{array}$$

$$\begin{array}{r|rrrrr} -4 & 1 & 6 & 10 & 8 & \\ & & -4 & -8 & -8 & \\ \hline & 1 & 2 & 2 & 0 & \end{array}$$

$$x^2 + 2x + 2 = 0$$

$$x^2 + 2x + 1 = -2 + 1$$

$$(x+1)^2 = -1$$

20.  $f(x) = x^4 + 3x^3 + 7x^2 + 15x + 10$ ;  $(x+2)$  is a factor

$$\begin{array}{r|rrrrr} -2 & 1 & 3 & 7 & 15 & 10 & \\ & & -2 & -2 & -10 & -10 & \\ \hline & 1 & 1 & 5 & 5 & 0 & \end{array}$$

$$x^3 + x^2 + 5x + 5 = 0$$

$$x^2(x+1) + 5(x+1) = 0$$

$$(x+1)(x^2 + 5) = 0$$

$$x = -1 \pm i, \pm 4$$

$$x^2 = -5$$

$$x = \pm i\sqrt{5}, 1, -2$$

\*\* Challenge:

$f(x) = 3x^4 - 2x^3 - 12x^2 + 6x + 9$ ;  $\pm\sqrt{3}$  are roots

Hint: start with  $x = \sqrt{3}$   $x = -\sqrt{3}$

$$\text{so } (x - \sqrt{3})(x + \sqrt{3}) = x^2 - 3$$

now use long division and quadratic formula

$$x = \frac{1 \pm \sqrt{10}}{3}, \pm\sqrt{3}$$