

#1-4. Write as an exponential equation:

1. $\log_4 \frac{1}{64} = -3$

$$4^{-3} = \frac{1}{64}$$

2. $\log_x 3 = 2$

$$x^2 = 3$$

3. $\ln 148 = 5$

$$\log_e 148 = 5$$
$$e^5 = 148$$

4. $\log x = 7$

$$10^7 = x$$

#5-7. Write as a logarithmic equation:

5. $13^2 = 169$

$$\log_{13} 169 = 2$$

6. $6^x = 3.6$

$$\log_6 3.6 = x$$

7. $e^3 = 20.1$

$$\ln 20.1 = 3 \text{ or } \log_e 20.1 = 3$$

#8-10. Expand using the properties of logs.

8. $\ln 2x^2y$

$$\ln 2 + \ln x^2 + \ln y$$

$$\ln 2 + 2\ln x + \ln y$$

9. $\log \frac{15x}{\sqrt{y}}$

$$\log 15 + \log x - \log y^{\frac{1}{2}}$$

$$\log 15 + \log x - \frac{1}{2} \log y$$

10. $\log_2 \frac{z}{x^3y}$

$$\log_2 z - \log_2 x^3 - \log_2 y$$

$$\log_2 z - 3\log_2 x - \log_2 y$$

$$\text{or } \log_2 z - (3\log_2 x + \log_2 y)$$

#11-13. Condense using the properties of logs.

11. $\log_8 5 + \log_8 3$

$$\log_8 15$$

12. $\frac{1}{3} \log x - \log 2$

$$\log x^{\frac{1}{3}} - \log 2$$

$$\log \frac{\sqrt[3]{x}}{2}$$

13. $\ln 3 - 4 \ln x - \frac{1}{2} \ln y$

$$\ln 3 - \ln x^4 - \ln \sqrt{y}$$

$$\ln \frac{3}{x^4 \sqrt{y}}$$

#14-17. Solve the following equations

14. $8^{2x-7} = 8^{x+4}$

$$2x-7 = x+4$$

$$x = 11$$

15. $2^{2x+3} = 16$

$$2^{2x+3} = 2^4$$

$$2x+3 = 4$$

$$2x = 1 \quad x = \frac{1}{2}$$

16. $4^x = \frac{1}{256}$

$$4^x = \frac{1}{4^4} \quad x = -4$$

$$4^x = 4^{-4}$$

17. $\left(\frac{1}{9}\right)^{x+1} = 27^{2x-4}$

$$\left(\frac{1}{3^2}\right)^{x+1} = 3^{3(2x-4)}$$

$$3^{-2(x+1)} = 3^{3(2x-4)}$$

$$-2x-2 = 6x-12$$

$$-8x = -10 \quad x = \frac{5}{4}$$

#18-27. Evaluate the following

18. $\log 10000 = x$

$$10^x = 10,000$$

$$10^x = 10^4 \quad x = 4$$

19. $\log_3 \frac{1}{243} = x$

$$3^x = \frac{1}{3^5} \quad 3^x = 3^{-5}$$

$$x = -5$$

20. $\log_{25} 5 = x$

$$25^x = 5^1$$

$$5^{2x} = 5^1$$

$$2x = 1 \quad x = \frac{1}{2}$$

21. $\log_9 1 = x$

$$9^x = 1$$

$$x = 0$$

22. $\log_5 5^4 = x$

$$x = 4$$

23. $\ln e^{1.7} = x$

$$x = 1.7$$

24. $\log_8 4 = x$

$$8^x = 4$$

$$2^{3x} = 2^2$$

$$3x = 2 \quad x = \frac{2}{3}$$

25. $\log_3 27 = x$

$$x = 3$$

26. $\log_4 32 = x$

$$4^x = 32$$

$$2^{2x} = 2^5$$

$$2x = 5$$

$$x = \frac{5}{2}$$

27. $\log_{\frac{1}{3}} 27 = x$

$$\left(\frac{1}{3}\right)^x = 27$$

$$-x = 3$$

$$3^{-x} = 3^3$$

$$x = -3$$

#28-30. Rewrite each using the change-of-base formula.

28. $\frac{\log_3 12}{\log_3 7}$ $\log_7 12$

29. $\frac{\log 15}{\log 2}$ $\log_2 15$

30. $\frac{\ln 5}{\ln 8}$ $\log_8 5$

#31-36. Solve the following log equations.

31. $\log_4 x = 3$

$4^3 = x$

$x = 64$

32. $\log_{64} x = \frac{1}{2}$

$64^{\frac{1}{2}} = x$

$x = \sqrt{64} = 8$

33. $\log_2 \left(\frac{1}{4}\right) = x$

$2^x = \frac{1}{4}$

$x = -2$

$2^x = 2^{-2}$

34. $\log_{\frac{1}{3}} x = -2$

$\left(\frac{1}{3}\right)^{-2} = x$

$x = 3^2 = 9$

35. $\log_x \frac{1}{4} = -2$

$x^{-2} = \frac{1}{4}$

$x^{-2} = 2^{-2}$

$x = 2$

36. $\log_{25} x = \frac{3}{2}$

$25^{\frac{3}{2}} = x$

$(\sqrt{25})^3 = x$

$x = 5^3 = 125$

#37-39. Find each inverse function.

37. $y = \ln(x-3) + 4$

$x = \ln(y-3) + 4$

$e^{x-4} = \ln(y-3)$

$e^{x-4} = y-3$

$y = e^{x-4} + 3$

38. $y = \log_2 x + 1$

$x = \log_2 y + 1$

$x-1 = \log_2 y$

$2^{x-1} = y$

39. $y = 4^x - 3$

$x = \log_4 y + 3$

$x+3 = \log_4 y$

$\log(x+3) = y \log 4$

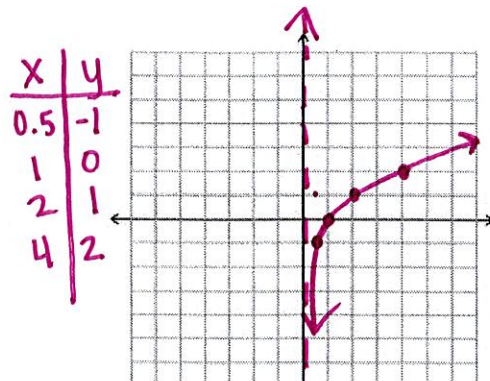
$y = \frac{\log(x+3)}{\log 4} = \log_4(x+3)$

#40-42. Graph each function.

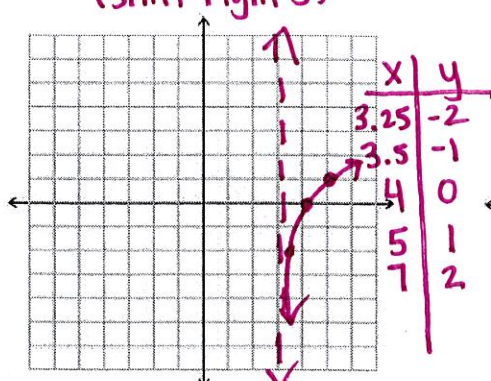
40. $y = \log_2 x$

41. $y = \log_2(x-3)$
(shift right 3)

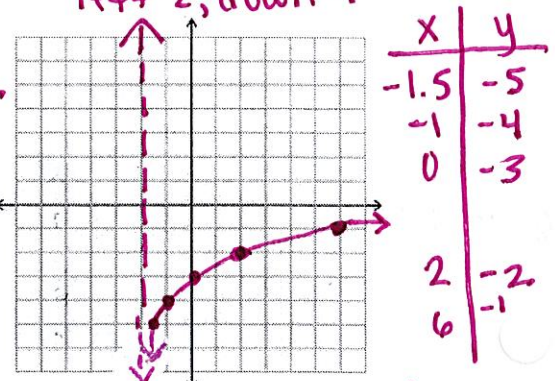
42. $y = \log_2(x+2) - 4$
left 2, down 4



inverse $\rightarrow x = \log_2 y$
 $y = 2^x$



inverse $\rightarrow x = \log_2(y-3)$
 $2^x = y-3$ $y = 2^x + 3$



inverse $\rightarrow x = \log_2(y+2) - 4$
 $x+4 = \log_2(y+2)$
 $2^{x+4} = y+2$ $y = 2^{x+4} - 2$

Solve the following log equations. Check for extraneous solutions!

1. $\log_4 x = 3$

$$4^3 = x$$

$$\boxed{x = 64}$$

2. $\log_3(2x+5) = 2$

$$3^2 = 2x+5$$

$$9 = 2x+5$$

$$2x = 4$$

$$\boxed{x = 2}$$

3. $\log_4(2x-1) = \log_4 16$

$$2x-1 = 16$$

$$2x = 17$$

$$\boxed{x = 17/2}$$

4. $\log_8(x^2-2) = \log_8 7$

$$x^2-2 = 7$$

$$\sqrt{x^2} = \sqrt{9}$$

$$\boxed{x = \pm 3}$$

5. $\log_2 4 + \log_2 6 = \log_2 x$

$$\log_2 24 = \log_2 x$$

$$\boxed{x = 24}$$

6. $\log_3 x - \log_3 12 = \log_3 3$

$$\log_3 \frac{x}{12} = \log_3 3$$

$$\frac{x}{12} = 3 \quad \boxed{x = 36}$$

7. $\log_5 x + \log_5 9 = \log_5 27$

$$\log_5 9x = \log_5 27$$

$$9x = 27$$

$$\boxed{x = 3}$$

8. $\log_3 4 - \log_3 x = 2$

$$\log_3 \frac{4}{x} = \log_3 3^2$$

$$x \cdot \frac{4}{x} = 9 \cdot x$$

$$9x = 4$$

$$\boxed{x = \frac{4}{9}}$$

9. $\log_9 5x = \log_9 6 + \log_9(x-2)$

$$\log_9 5x = \log_9 6(x-2)$$

$$5x = 6x - 12$$

$$-x = -12$$

$$\boxed{x = 12}$$

10. $\log x = 2 \log 3 + \frac{1}{3} \log 8$

$$\log x = \log 3^2 + \log 8^{\frac{1}{3}}$$

$$\log x = \log 9 + \log \sqrt[3]{8} = 2$$

$$\log x = \log 9 \cdot 2$$

$$\log x = \log 18$$

$$\boxed{x = 18}$$

11. $\log_2 x + \log_2(x+3) = 2$

$$\log_2 x(x+3) = \log_2 2^2$$

$$x^2 + 3x = 4$$

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$x+4 = 0 \quad x = -4$$

$$x-1 = 0 \quad \boxed{x = 1}$$

12. $\log_7(2x+1) - \log_7 5 = \log_7 3$

$$\log_7 \frac{2x+1}{5} = \log_7 3$$

$$\frac{2x+1}{5} = 3$$

$$2x+1 = 15$$

$$2x = 14$$

$$\boxed{x = 7}$$

Solve the following exponential equations. Round your answers to 3 decimal places.

13. $4.6^x = 32.1$

$$\log 4.6^x = \log 32.1$$

$$x \frac{\log 4.6}{\log 4.6} = \frac{\log 32.1}{\log 4.6}$$

$$x \approx 2.273$$

14. $3^{5x} = 8$

$$\ln 3^{5x} = \ln 8$$

$$5x \frac{\ln 3}{\ln 3} = \frac{\ln 8}{\ln 3}$$

$$5x = 1.893$$

$$x \approx 0.379$$

15. $7^{x+1} = 12$

$$\log 7^{x+1} = \log 12$$

$$(x+1) \frac{\log 7}{\log 7} = \frac{\log 12}{\log 7}$$

$$x+1 = 1.277$$

$$x \approx 0.277$$

16. $e^{x-2} = 11$

$$\ln e^{x-2} = \ln 11$$

$$x-2 = \ln 11 + 2$$

$$x \approx 4.398$$

17. $\frac{125}{5} = \frac{5e^{12x}}{5}$

$$e^{12x} = 25$$

$$\ln e^{12x} = \ln 25$$

$$12x = \frac{\ln 25}{12}$$

$$x \approx 0.268$$

18. $3e^{x+2} - 4 = 10$

$$3e^{x+2} = 14$$

$$e^{x+2} = \frac{14}{3}$$

$$\ln e^{x+2} = \ln \left(\frac{14}{3}\right)$$

$$x+2 = 1.540$$

$$x \approx -0.460$$

Applications.

19. You deposit \$2500 into an account that pays 3.5% annual interest compounded daily. How long will it take for the balance to reach \$3000?

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$\frac{3000}{2500} = \frac{2500}{2500} \left(1 + \frac{0.035}{365}\right)^{365t}$$

$$1.2 = \left(1 + \frac{0.035}{365}\right)^{365t}$$

$$\frac{\ln 1.2}{\ln \left(1 + \frac{0.035}{365}\right)} = \frac{365t \cdot \ln \left(1 + \frac{0.035}{365}\right)}{\ln \left(1 + \frac{0.035}{365}\right)}$$

20. An initial deposit of \$4000 is made in a savings account for which the interest is compounded continuously. The balance will triple in 15 years. What is the annual rate of interest for this account?

$$A = Pe^{rt}$$

$$\frac{12000}{4000} = \frac{4000}{4000} e^{15r}$$

$$e^{15r} = 3$$

$$\ln e^{15r} = \ln 3$$

$$15r = \frac{\ln 3}{15}$$

$$r \approx 0.073 \approx 7.3\%$$

$$\frac{365t}{365} = \frac{1901.495}{365}$$

$$t \approx 5.2 \text{ years}$$

21. The yield V (in millions of cubic feet per acre) for a forest at age t years is given by $V = 6.7e^{-\frac{48.1}{t}}$. Find the time necessary to have a yield of 1.7 million cubic feet.

$$\frac{1.7}{6.7} = \frac{6.7e^{-\frac{48.1}{t}}}{6.7}$$

$$0.254 = e^{-\frac{48.1}{t}}$$

$$\ln 0.254 = \ln e^{-\frac{48.1}{t}}$$

$$\ln 0.254 = -\frac{48.1}{t}$$

$$t \cdot \frac{\ln 0.254}{\ln 0.254} = \frac{-48.1}{\ln 0.254}$$

$$t \approx 35.1 \text{ years}$$

LOGS REVIEW WS 2 - ANSWERS

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|---------------|------------------|-------------------|------------|-----------|------------|
| 1. 64 | 2. 2 | 3. $\frac{17}{2}$ | 4. ± 3 | 5. 24 | 6. 36 |
| 7. 3 | 8. $\frac{4}{9}$ | 9. 12 | 10. 18 | 11. 1 | 12. 7 |
| 13. 2.273 | 14. 0.379 | 15. 0.277 | 16. 4.398 | 17. 0.268 | 18. -0.460 |
| 19. 5.3 years | 20. 7.3% | 21. 35.07 years | | | |