NORMAL DISTRIBUTIONS

STANDARD AND NON-STANDARD

Z - SCORE

- When the data value in question does not fall on exactly one, two, or three standard deviations from the mean, we can no longer rely on The Empirical Rule.

- What do we do?

- Use z-Score!

$$z = \frac{x - \bar{x}}{s}$$

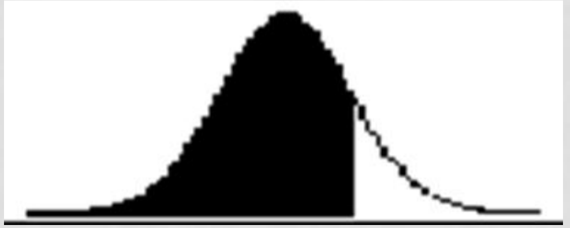
Z-SCORES

- Find the z-score and use a z-table to determine the probability that falls BELOW that data value.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015

Z-SCORES

- <u>Example 1</u>: Find the probability a data value falls below 1.15 standard deviations from the mean.



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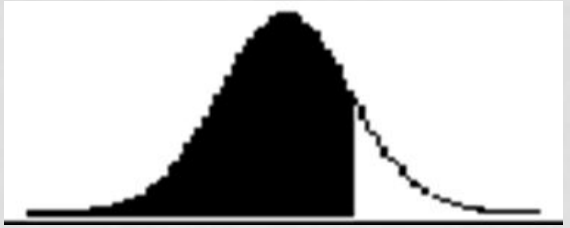
Z-SCORES

- <u>Example 2</u>: Find the probability a data value falls below 0.82 standard deviations from the mean.



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EXAMPLE

- Example 3: A normal distribution has a mean of 70 and a standard deviation of 10. Find the probability that a randomly selected data value from the distribution is in the given interval. Draw a sketch to represent each interval. Use your z-table for probabilities.

  - A) $P(x \le 65)$

  - B) P (x □ 47)

  - C) $P(39 \le x \le 82)$

Answers: A) 0.3085 B) 0.9893 C) 0.8839

EXAMPLE

- Example 4: The data for the SAT is normally distributed with a mean of 1000 and standard deviation 180.

(a) What percent of students score under 1200?

Find z-score: $$z = \frac{x - \overline{x}}{\sigma} = \frac{1200 - 1000}{180} \approx 1.11$$

Use z-table to get probability: $$P(\text{x} \leq 1200) = .8665$$

Approximately 86.65% of testers score under 1200.

EXAMPLE

(b) What percent of test takers score ABOVE 1200?

Use z-table to get probability: $P(\text{x} \le 1200) = .8665$

Subtract the probability from 1.

$$P(\text{x} \ge 1200) = 1 - 0.8665 = 0.1335$$

Approximately 13.35% of testers score above1200.

EXAMPLE

(c) What is the probability that a student scores between 900 and 1300?

Find z-scores: $z_1 = \dfrac{900-1000}{180} \approx -0.56$ $z_2 = \dfrac{1300-1000}{180} \approx 1.67$

Use z-table: $P_1 = 0.2877$ $P_2 = 0.9525$

Subtract probabilities: $0.9525 - 0.2877 = 0.6648$

(Approximately 66.48% of testers score between 900 and 1300.)

STANDARD VS. NON-STANDARD

- The previous examples are considered non-standard normal distribution.

- For a standard normal distribution, the mean is 0 and the standard deviation is 1.