GSE Honors Algebra II
Unit 6 Test Review

Name Key!
Date __________ Period _____

Normal Distributions and the Empirical Rule

For the Empirical Rule…

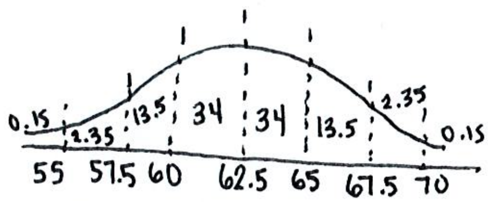
- 68% of all observations fall within one standard deviation of the mean
- 95% of all observations fall within two standard deviations of the mean
- 99.7% of all observations fall within three standard deviations of the mean10

1) A normal distribution has a mean of 28 and a standard deviation of 6. Find the probability that a randomly selected x value from the distribution is in the interval from 16 to 40.

   16 22 28 34 40

   95%] percent 0.95] probability

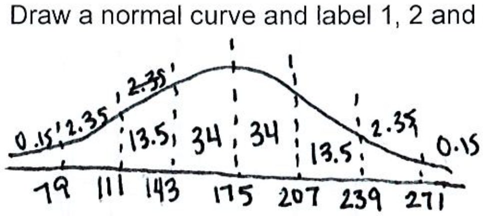
2) The distribution of heights of young women aged 18 to 24 is approximately normal with mean 62.5 inches and *s* 2.5 inches. Use the Empirical Rule
   a. Draw a normal distribution for this data set, showing the Empirical Rule. Use the Empirical rule for the remaining questions.



   b. What percent of women are taller than 67.5 in? $2.35 + 0.15 = \boxed{2.5\%}$
   c. Between what heights do the middle 95% of women fall? $\boxed{57.5 - 67.5}$
   d. What percent of women are shorter than 5 feet?
      5 ft = 60 in
      $13.5 + 2.35 + 0.15 = \boxed{16\%}$
   e. A height of 65 inches corresponds to what percentile of adult female (18-24) heights?
      100 - 16 = $\boxed{\text{84th percentile}}$

3) Given an approximately normal distribution with a mean of 175 and a standard deviation of 32….
   (Use the Empirical Rule)

   a. Draw a normal curve and label 1, 2 and 3 standard deviations from the mean.

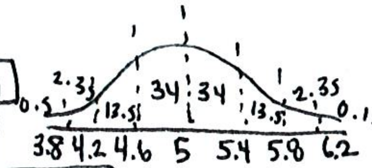


   b. What percent of values are within the interval (143, 207)?
      $\boxed{68\%}$
   c. What percent of values are within the interval (79, 143)?
      $2.35 + 13.5 = \boxed{15.85\%}$
   d. What percent of values are outside the interval (111, 239)?
      $2.5 + 2.5 = \boxed{5\%}$
   e. What percent of values are $\underbrace{\text{less than 143}}_{16\%}$ or $\underbrace{\text{greater than 271?}}_{0.15\%}$ $\boxed{16.15\%}$

4) The weights of 1500 fish in a lake are normally distributed with a mean of 5kg and a standard deviation of 0.4kg. Use the Empirical Rule.

a. About how many fish weigh 4.6 kg or more? $0.84(1500)$

$34+34+13.5+2.35+0.15=84\%$ $=\boxed{1260 \text{ fish}}$



b. About how many fish weigh less than 4.2 kg?

$2.35+0.15=2.5\%$ $0.025(1500)=\boxed{38 \text{ fish}}$

c. About how many fish weigh between 4.6 kg and 5.8 kg?

$34+34+13.5=81.5\%$ $0.815(1500)=\boxed{1223 \text{ fish}}$

d. About how many fish weigh between 4.2 kg and 6.2 kg?

$13.5+68+13.5+2.35=97.35$ $0.9735(1500)=\boxed{1460 \text{ fish}}$

5) A park ranger samples 27 trees in a wooded area and found that the mean diameter of the trees is 15.2 inches with a standard deviation of 3.5 inches. Suppose that this sample of trees provides an accurate description of the entire forest and that the trees diameters are normally distributed.

See bell curve next to #6

a. What range of diameters would encompass the middle 95% of trees? $\boxed{(8.2, 22.2)}$

b. What PERCENT of the trees in the forest would you expect to be under 8.2 inches in diameter?

$2.35+0.15=\boxed{2.5\%}$

c. What is the PROBABILITY that a randomly selected tree will be between 11.7 and 15.2 inches in diameter? decimal $\boxed{0.34}$

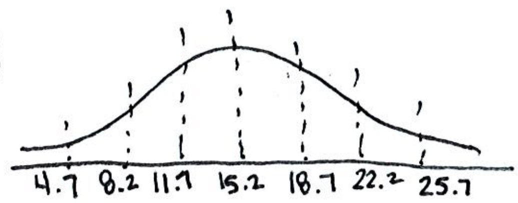
d. If there are 1, 540 trees in the park, about HOW MANY trees are more than 18.7 inches in diameter?

$13.5+2.35+0.15=16\%$ $0.16(1540)\approx\boxed{246 \text{ trees}}$

Normal Distributions and Z-Scores
(Use your Z-Table)

6) A normal distribution has a mean of 90 and a standard deviation of 13. Find the z-score for each data point given below.

#5



a. 76 $z=\dfrac{76-90}{13}\approx\boxed{-1.08}$

b. 96 $z=\dfrac{96-90}{13}\approx\boxed{0.46}$

c. 120 $z=\dfrac{120-90}{13}\approx\boxed{2.31}$

d. 59 $z=\dfrac{59-90}{13}\approx\boxed{-2.38}$

7) On one measure of attractiveness, scores are normally distributed with a mean of 5.9 and a standard deviation of 0.6.

a. What percent of the population would be rated 7.0 or better? to right

$z=\dfrac{7-5.9}{0.6}\approx 1.83$

$1-0.9664=$ $.0336$ $=\boxed{3.36\%}$

b. Find $P(x\le 5.0)$ to left $z=\dfrac{5-5.9}{0.6}=-1.5\rightarrow\boxed{0.0668}$

c. Find $P(6.1\le x\le 7.5)$

$z=\dfrac{6.1-5.9}{0.6}\approx 0.33$ $z=\dfrac{7.5-5.9}{0.6}=2.67$ $0.9236-0.6293$ $\boxed{=0.2943}$

8) Scores on an anti-aircraft exam are normally distributed with a mean of 99.6 and a standard deviation of 24.7. For a randomly selected subject, find the probability that a score will fall between a 105 and a 135.

$z=\dfrac{105-99.6}{24.7}=0.22\rightarrow 0.5871$ $z=\dfrac{135-99.6}{24.7}\approx 1.43\rightarrow 0.9236$

$0.9236-0.5871=\boxed{0.3365}$

9) For a certain population, scores on the Miller Analogies Test are normally distributed with a mean of 58.7 and a standard deviation of 15.9. If subjects who score under a 28.00 are to be given special training, what percentage of subjects will get special training?

$z=\dfrac{28-58.7}{15.9}\approx -1.93\rightarrow 0.0268\approx\boxed{2.68\%}$

$\boxed{\approx 24 \text{ people}}$

Last step → 0.0444(545)

10) Scores on the biology portion of the Medical College Admissions Test (MCAT) are normally distributed with a mean of 8.0 and a standard deviation of 1.6. If 545 students take the test, how many are expected to score above a 11.5 or below a 5.0?

$z = \frac{5-8}{1.6} \approx -1.88 \to 0.0301$

$z = \frac{11.5-8}{1.6} \approx 2.19 \to 0.9857$

0.0444 ↑

0.0301 + 0.0143

11) The Brake Stop wants to offer a guarantee to its customers that its brake pads will last for a certain number of miles. They find their brake pads last an average of 40,000 miles with a standard deviation of 3,580 miles. They want to guarantee their brake pads so that only 2% of customers need to have the pads replaced before the warranty expires. How many miles should they guarantee their brake pads for?

2% = 0.02

What is the z-score with a prob. of 0.02 → z = −2.05

$-2.05 = \frac{x - 40{,}000}{3580}$

$x - 40{,}000 = -7339$

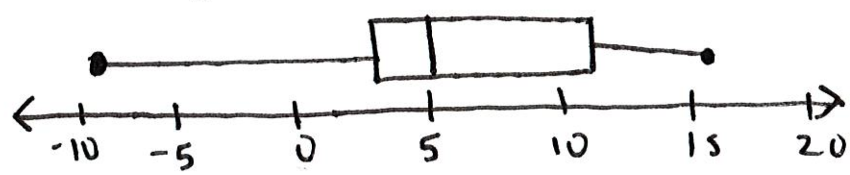
$\boxed{x = 32{,}661 \text{ miles}}$

12) Using the data provided, please find the following information:

5, -8, 10, 3, 11, 12, 16, 4, 5, 1, 16

a) 5 number summary: min: −8 Q_2(median): 5 max: 16
Q_1: 3 Q_3: 12

b) Box and Whisker Plot:



c) Variance $\boxed{50.55}$

d) Standard Deviation $\boxed{7.11}$