Honors Geometry
Triangle Centers Constructions
Incenter, Centroid,
Circumcenter & Orthocenter

MCC9-12.G.CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

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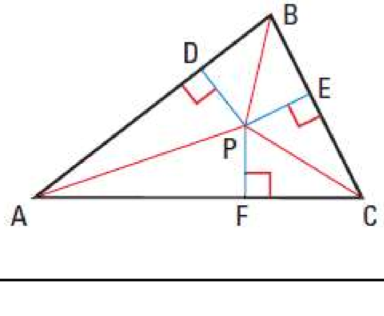
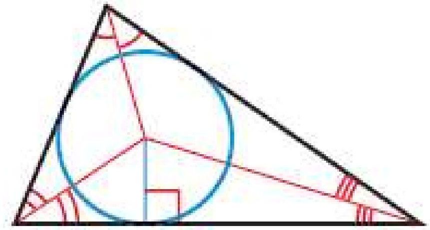
Incenter Properties

- The three **angle bisectors** of a triangle intersect at the **incenter**.
- The incenter is **equidistant** from the sides of the triangle.
- Meaning, if you make a perpendicular segment from the incenter to each side of the triangle, the three perpendicular segments are of equal length.
- The incenter is **always** inside the triangle.
- If you draw a circle using the incenter as the center of the circle and a perpendicular segment as the radius, the result is an inscribed circle called the **incircle**.





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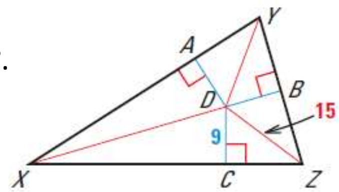
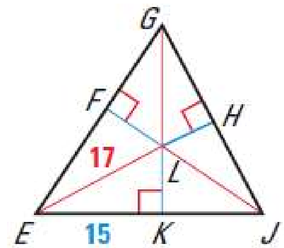
Examples

- In the diagram, *D* is the incenter of ΔXYZ. Find *DB*.



- In the diagram, *L* is the incenter of ΔEGJ. Find *HL*.



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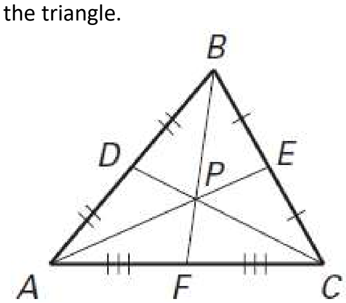
Centroid Properties

- **Median:** A segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.
- The three medians of a triangle intersect at the **centroid**.
- The centroid is two-thirds of the distance from each vertex to the midpoint of the opposite side.
- Meaning, the segment from the vertex to the centroid is twice as long as the segment from the centroid to the midpoint.
- The centroid is **always** inside the triangle.



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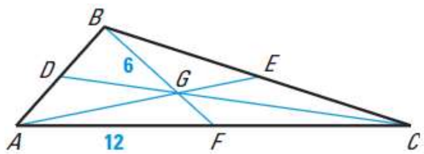
Example

- Point G is the centroid of ΔABC and BG = 6, AF = 12, and AE = 15.



FC = _________ GF = _________

BF = __________ AG = __________

GE = __________

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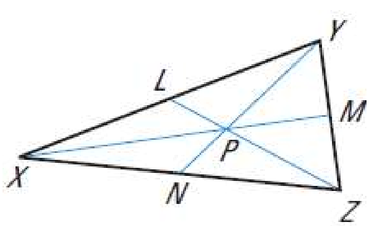
Example

- The medians of ΔXYZ intersect at point P, YP = 12, LX = 15, and LZ = 18.



LY = _________ NP = _________

YN = __________ ZP = __________

LP = __________

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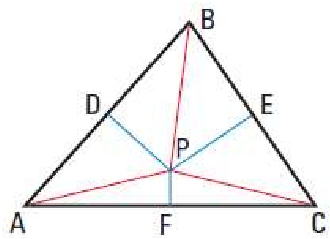
Circumcenter Properties

- **Perpendicular Bisector:** A line, ray, or segment perpendicular to a segment at its midpoint.
- The three perpendicular bisectors of a triangle intersect at the **circumcenter**.
- The circumcenter is equidistant from the vertices of the triangle.
- Meaning, if you make a segment from the circumcenter to each vertex of the triangle, the three segments are of equal length.



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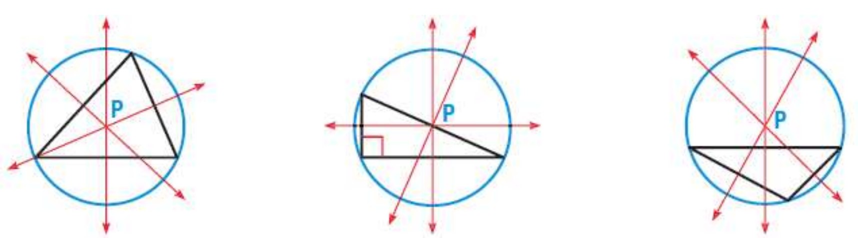
Circumcenter Properties (Cont.)

- The circumcenter can fall inside of, on, or outside of the triangle.
- If you draw a circle using the circumcenter as the center of the circle and a segment from the circumcenter to a vertex as the radius, the result is an circumscribed circle called the **circumcircle**.



Acute triangle
P is inside triangle.

Right triangle
P is on triangle.

Obtuse triangle
P is outside triangle.

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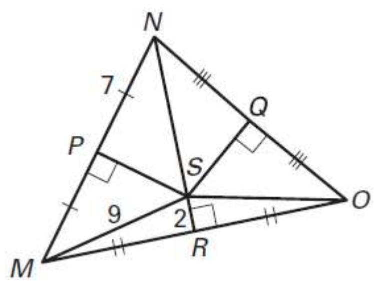
Example

- The perpendicular bisectors of ΔMNO meet at point S. Find:



NS = OS =

PM =

OR =

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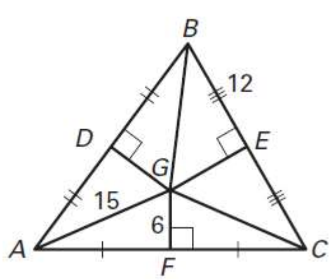
Example

- G is the circumcenter of ΔABC. Find the following:



BG = CG =

EC =

CF =

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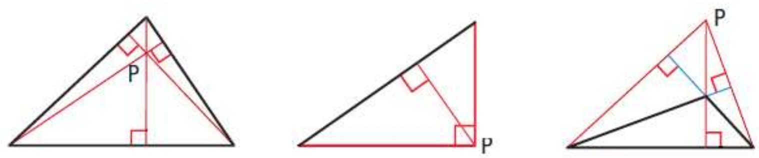
Orthocenter Properties

- **Altitude:** The perpendicular segment from a vertex of a triangle to the opposite side or to the line containing the opposite side.
- The three perpendicular altitudes of a triangle intersect at the **orthocenter**.
- The orthocenter can fall inside of, on, or outside of the triangle.



Acute triangle
P is inside triangle.

Right triangle
P is on triangle.

Obtuse triangle
P is outside triangle.

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Fun Facts

- Fun fact #1: The centroid, circumcenter, and orthocenter always lie on a line called the Euler line.
- Fun fact #2: If the triangle is equilateral, then the circumcenter, centroid, and orthocenter are all the same point.

- http://www.mathopenref.com/eulerline.html

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