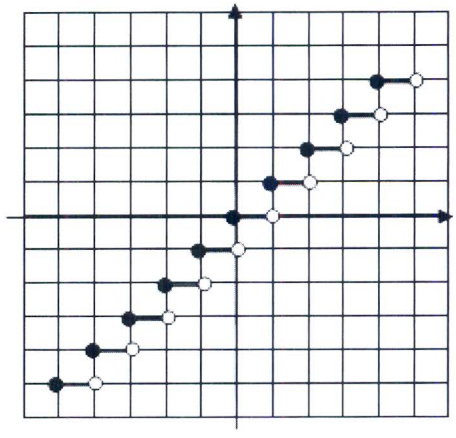
Investigating Step Functions Name Fuston

Part 1: Don't Give Me Fractions...



If you ask someone their age, they seldom tell the truth. Although you might have been alive for 14 years, 6 months, 9 hours and 34 minutes, you'll probably say you're 14 years old. You are using the **greatest integer function** or f(x) = [x]. Its technical definition is "the greatest integer less than or equal to x", but most people think of it as the "floor function" or "round down function", because that's what happens to x.

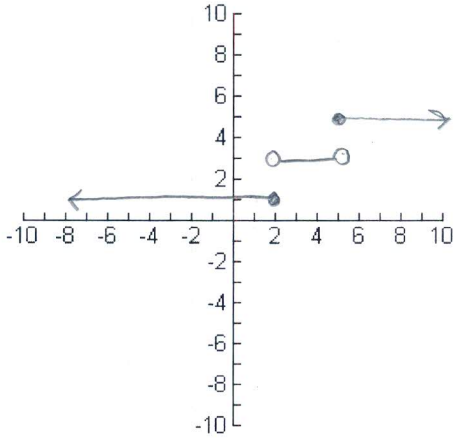
What is [6]? 6 [5.9] = 5 [−3.4] = −4

Its graph appears to be a series of stair steps. Why is this so?

(answers vary - be creative!)

Part 2: Let's graph another step function ...

Graph $f(x) = \begin{cases} 5 & x \geq 5 \\ 3 & 2 < x < 5 \\ +1 & x \leq 2 \end{cases}$



Domain: $(-\infty, \infty)$

Range: $\{1, 3, 5\}$

Points of Discontinuity: $x = 2, x = 5$

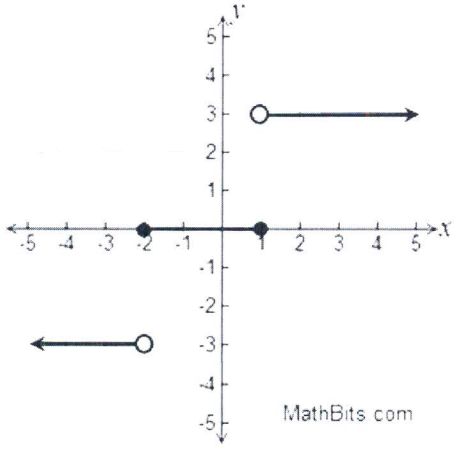
Intervals Constant: $(-\infty, 2)(2, 5)(5, \infty)$

Intervals of increasing: None

Part 3: Write the piecewise function ...

Write the step function as a piecewise function.

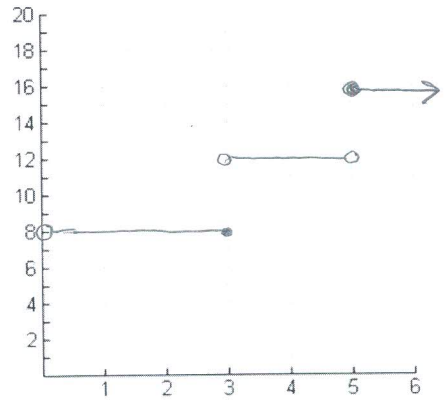
$f(x) = \begin{cases} -3, & x < -2 \\ 0, & -2 \leq x \leq 1 \\ 3, & x > 1 \end{cases}$



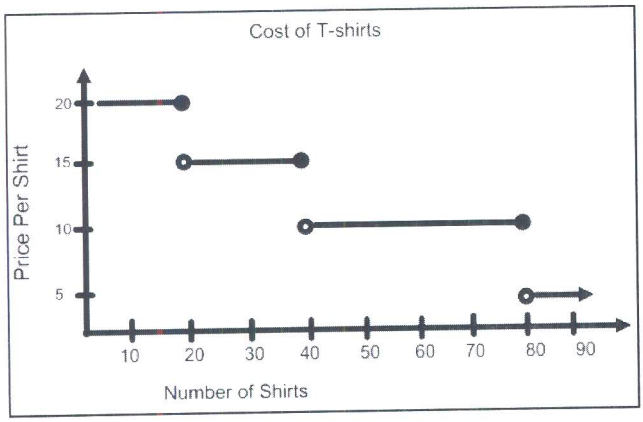
Part 4: Applications ...

You get paid to pick up recycling materials along the side of the road every day. You are given a bag each day to collect plastic bottles and aluminum cans in. If you collect 3 pounds or less than 3 pounds, you earn \$8. If you collect between 3 and 5 pounds, you get \$12 and if you collect 5 pounds or more than 5 pounds, you get \$16. Write a piecewise function and a graph that models your income.

$$f(x) = \begin{cases} \$8, & 0 < x \leq 3 \\ \$12, & 3 < x < 5 \\ \$16, & x \geq 5 \end{cases}$$



The following application rounds up instead of down. In ~~the first~~ this example, the cost of t-shirts decreases per shirt as the number of shirts ordered increases.



a. If your club orders 40 t-shirts, what is the cost per shirt?

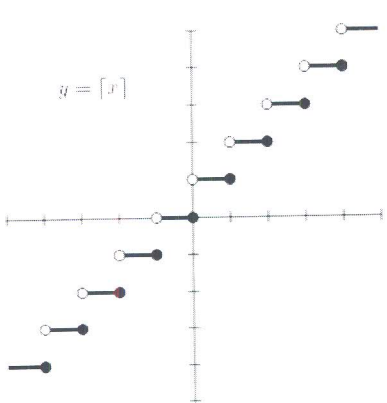
\$15

b. If your club orders 41 t-shirts, what is the cost per shirt?

\$10

c. Write the step function as a piecewise function.

$$f(x) = \begin{cases} \$20, & 0 < x \leq 20 \\ \$15, & 20 < x \leq 40 \\ \$10, & 40 < x \leq 80 \\ \$5, & x > 80 \end{cases}$$



Part 5: Least integer function ...

This is a graph of the **least integer function** or f(x) = $\lceil x \rceil$. Its technical definition is "the least integer greater than or equal to x", but most people think of it as the "ceiling function" or "round up function", because that's what happens to x.

The t-shirt example above followed this notion. Can you think of other places in real life that follow either a greatest integer or a least integer model? ① the price of postage