Inverse Functions – A Graphical Approach

A relation is called a function when each input (x) maps to only one output (y). (no x-coordinates repeat)

If a graph passes the vertical line test, the relation is a function. (no x-coordinates repeat)

Inverses: Two functions are inverses, if and only if, when one function contains a point (a,b), the other function contains the point (b, a)

Example: f(x): {(3,1) , (-2,4) , (5,-1) }

x Domain of f(x) {3, -2, 5}

y Range of f(x) {1, 4, -1}

f(x)

The **inverse** of f(x), written as $f^{-1}(x)$, will be { (1,3) (4,-2) (-1,5) }

x Domain of inverse {1, 4, -1}

y Range of inverse {3, -2, 5}

$f^{-1}(x)$

***The domain of function f(x) has become the range of the inverse;

The range of function f(x) has become the domain of the inverse.

Example 1: Tables

x	y
0	-4
1	-1
2	2
3	5

a. Is the original relation a function? yes! x's don't repeat

b. Write the inverse.

c. Is the inverse a function? yes! x's don't repeat

$f^{-1}(x)$

x	y
-4	0
-1	1
2	2
5	3

Example 2: Graphs

f(x)

x	y
0	-4
1	-1
2	2
3	5

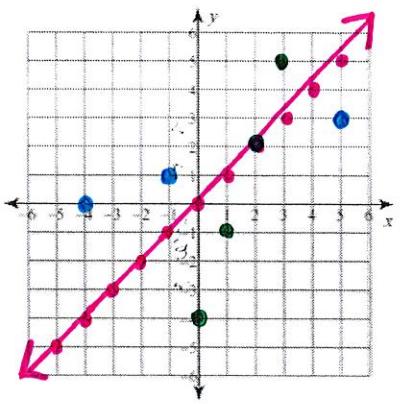
a. Graph the original relation in one color.

b. Graph the inverse in a different color.

c. Graph y=x y = 1x + 0

d. What do you notice?

y=x is a line of symmetry



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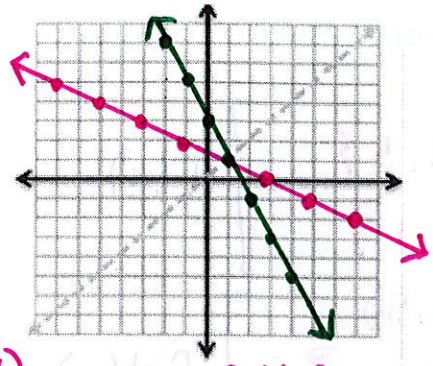
Example 3: More graphs!

Graph each function and its inverse.

f(x)

a. y = -2x +3



x	y
-2	7
-1	5
0	3
1	1
2	-1

$f^{-1}(x)$

x	y
7	-2
5	-1
3	0
1	1
-1	2

$f^{-1}(x)$

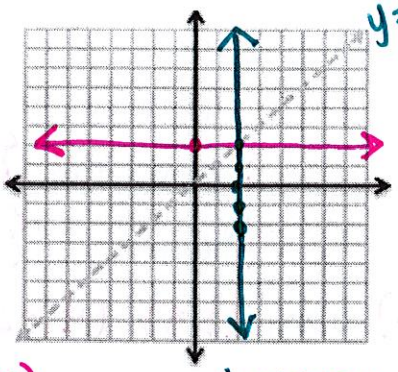
$x = -2y + 3$

$\frac{x}{-2} - \frac{3}{-2} = \frac{-2y}{-2}$

$y = \frac{-1}{2}x + \frac{3}{2}$

$y = 0x + 2$

b. y = 2



f(x)

x	y
-2	2
-1	2
0	2
1	2
2	2

function!

$f(x) = 2$

$y = 2$

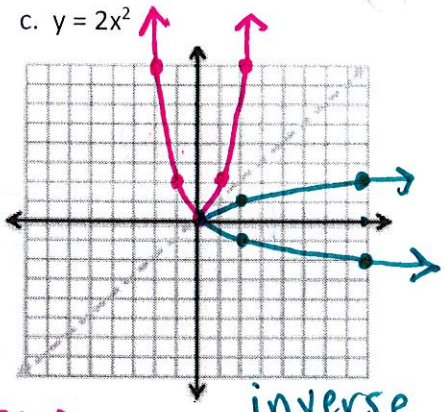
inverse

x	y
2	-2
2	-1
2	0
2	1
2	2

not a function!

$x = 2$

c. $y = 2x^2$



f(x)

x	y
-2	8
-1	2
0	0
1	2
2	8

function!

inverse

x	y
8	-2
2	-1
0	0
2	1
8	2

not a function?

$y = 2x^2$

$\frac{x}{2} = \frac{2y^2}{2}$

$\sqrt{y^2} = \frac{\sqrt{x}}{\sqrt{2}}$

$y = \frac{\sqrt{x}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$y = \frac{\pm\sqrt{2x}}{2}$

Things to know....

- The inverse of a function is denoted by $f^{-1}(x)$
- To graph in inverse: Make a table, switch x and y, re-graph
- The inverse of a graph is reflected over the line y=x
- A **one-to-one function** is a special type of function where the original and inverse are BOTH functions (original function will pass the horizontal line test as well)