

**GSE Algebra II**  
**Unit 3 – Review on Polynomials**

Name: Key  
 Date: \_\_\_\_\_ Period: \_\_\_\_\_

Using your calculator and synthetic division, find ALL the roots of the following polynomial functions.

1.  $6x^4 - 11x^3 - 29x^2 + 19x + 15$  *Graphing Calc. to find rational zeros*

$$\begin{array}{r|rrrrr} 1 & 6 & -11 & -29 & 19 & 15 \\ & \downarrow & & & & \\ 3 & 6 & -5 & -34 & -15 & 0 \\ & \downarrow & & & & \\ & 6 & 13 & 5 & 0 & \end{array}$$

$$6x^2 + 13x + 5 = 0$$

$$(3x+5)(2x+1) = 0$$

$X = \{1, 3, -5/3, -1/2\}$

2.  $2x^4 + x^3 - 11x^2 + 11x - 3$

$$\begin{array}{r|rrrrr} -3 & 2 & 1 & -11 & 11 & -3 \\ & \downarrow & & & & \\ 1 & 2 & -5 & 4 & -1 & 0 \\ & \downarrow & & & & \\ & 2 & -3 & 1 & 0 & \end{array}$$

$$2x^2 - 3x + 1 = 0$$

$$(2x-1)(x-1) = 0$$

$X = \{-3, 1, 1/2\}$

3.  $x^4 + 5x^2 - 36$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 5 & 0 & -36 \\ & \downarrow & & & & \\ -2 & 1 & 2 & 9 & 18 & 0 \\ & \downarrow & & & & \\ & 1 & 0 & 9 & 0 & \end{array}$$

$$x^2 + 9 = 0$$

$$x = \pm 3i$$

$X = \{-2, 2, \pm 3i\}$

4.  $4x^4 + 32x^3 + 85x^2 + 93x + 36$

$$\begin{array}{r|rrrrr} -4 & 4 & 32 & 85 & 93 & 36 \\ & \downarrow & & & & \\ -1 & 4 & 16 & 21 & 9 & 0 \\ & \downarrow & & & & \\ & 4 & 12 & 9 & 0 & \end{array}$$

$$4x^2 + 12x + 9 = 0$$

$$(2x+3)(2x+3) = 0$$

$X = \{-4, -1, -3/2\}$

5.  $x^4 - 30x^2 - 88x - 315$

$$\begin{array}{r|rrrrr} -5 & 1 & 0 & -30 & -88 & -315 \\ & \downarrow & & & & \\ 7 & 1 & -5 & -5 & -63 & 0 \\ & \downarrow & & & & \\ & 1 & 2 & 9 & 0 & \end{array}$$

$$x^2 + 2x + 9 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(9)}}{2(1)} = \frac{-2 \pm \sqrt{-32}}{2}$$

$X = \{-5, 7, 1 \pm 2i\sqrt{2}\}$

6.  $x^4 - 4x^3 - 5x^2 + 18x - 90$

$$\begin{array}{r|rrrrr} -3 & 1 & -4 & -5 & 18 & -90 \\ & \downarrow & & & & \\ 5 & 1 & -7 & 16 & -30 & 0 \\ & \downarrow & & & & \\ & 1 & -2 & 6 & 0 & \end{array}$$

$$x^2 - 2x + 6 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(6)}}{2(1)} = \frac{2 \pm \sqrt{-20}}{2}$$

$X = \{3, 5, 1 \pm i\sqrt{5}\}$

Find all the zeros of the functions:

7.  $x^3 - 6x^2 - 7x + 60$ , given 4 is a solution

$$\begin{array}{r|rrrr} 4 & 1 & -6 & -7 & 60 \\ & \downarrow & & & \\ & 1 & -2 & -15 & 0 \end{array}$$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$X = \{-3, 4, 5\}$

8.  $x^3 - 5x^2 - 48x + 108$  given  $f(2) = 0$

$$\begin{array}{r|rrrr} 2 & 1 & -5 & -48 & 108 \\ & \downarrow & & & \\ & 1 & -3 & -54 & 0 \end{array}$$

$$x^2 - 3x - 54 = 0$$

$$(x-9)(x+6) = 0$$

$X = \{-6, 2, 9\}$

Use the following information to answer the questions.

9. Given  $f(x) = x^3 - 6x^2 + 3x + 10$

a. How many solutions will this function have? How do you know? 3 cubic

b. find  $f(3)$ . Write your answer as a coordinate pair.  $\begin{array}{r} 3 \overline{) 1 \ -6 \ 3 \ -10} \\ \underline{3 \ -9 \ -6} \\ 1 \ -3 \ -6 \ \underline{18} \end{array}$   $f(3) = 8$   $(3, 8)$

c. divide  $f(x)$  by  $(x+1)$ . Is it a factor of  $f(x)$ ? Why or why not?

$\begin{array}{r} -1 \overline{) 1 \ -6 \ 3 \ 10} \\ \underline{-1 \ 7 \ -10} \\ 1 \ -7 \ 10 \ \underline{0} \end{array}$  Yes, remainder = 0

d. divide  $f(x)$  by  $(x-1)$  Is it a factor of  $f(x)$ ? Why or why not?

$\begin{array}{r} 1 \overline{) 1 \ -6 \ 3 \ 10} \\ \underline{1 \ -5 \ 2} \\ 1 \ -5 \ -2 \ \underline{18} \end{array}$  No, remainder  $\neq 0$

e. find  $f(-1)$ . Write your answer as a coordinate pair. What does this coordinate pair represent?

$f(-1) = 0$   $(-1, 0)$  zero, x-int, root, solution

f. Find the rest of the zero's of the function.

$x = \{-1, 2, 5\}$

g. find  $f(0)$ . Write your answer as a coordinate pair. What does this coordinate pair represent?

$f(0) = 10$   $(0, 10)$  y-int.

10. Given  $f(x) = x^4 - 2x^3 - 3x^2 - 10x - 40$

a. How many solutions will this function have? How do you know? 4

b. find  $f(5)$ . Write your answer as a coordinate pair.  $f(5) = 210$   $(5, 210)$

c. divide  $f(x)$  by  $(x-4)$ . Is it a factor of  $f(x)$ ? Why or why not?

$\begin{array}{r} 4 \overline{) 1 \ -2 \ -3 \ -10 \ -40} \\ \underline{4 \ 8 \ 20 \ 40} \\ 1 \ 2 \ 5 \ 10 \ \underline{0} \end{array}$  Yes, remainder  $\emptyset$

d. find  $f(-2)$  Is it a factor of  $f(x)$ ? Why or why not?

$f(-2) = 0$  Yes

e. find the y-intercept of the function

$f(0) = -40$

f. Find the rest of the zero's of the function.

$\begin{array}{r} -2 \overline{) 1 \ 2 \ 5 \ 10} \\ \underline{-2 \ 0 \ -10} \\ 1 \ 0 \ 5 \ \underline{0} \end{array}$   $x = \{+i\sqrt{5}, -2, 5\}$

11. Given a polynomial  $h(x)$  where  $h(1) = 4$ ,  $h(-2) = 0$ ,  $h(-4) = -2$ ,  $h(0) = 0$ ,  $h(5) = 0$ ,  $h(1) = -2$

a. What is the least possible degree of this polynomial? 3

b. Identify any point on  $h(x)$  located in quadrant I (1, 4)

c. What is the y-intercept of  $h(x)$ ? (0, 0)

d. What are the real zero's of the function  $h(x)$ ?  $x = -2, 0, 5$

e. What are the factors of the function  $h(x)$ ?  $(x+2)(x)(x-5)$

f. What is a possible equation for  $h(x)$ ?  $= a(x)(x+2)(x-5)$

12. One factor of  $x^3 - 4x^2 + x + 6$  is  $x-3$ . Find the other factors AND find all the roots.

$$\begin{array}{r|rrrr} 3 & 1 & -4 & 1 & 6 \\ & \downarrow & & & \\ & & 3 & -3 & -6 \\ \hline & 1 & -1 & -2 & 0 \end{array} \quad x^2 - x - 2 \rightarrow (x-2)(x+1)(x-3)$$

$$x = 2, -1, 3$$

13. Given that  $x+2$  is a factor of  $2x^3 - x^2 - 7x + 6$ , factor to find all the zeros.

$$\begin{array}{r|rrrr} -2 & 2 & -1 & -7 & 6 \\ & \downarrow & & & \\ & & -4 & 10 & -6 \\ \hline & 2 & -5 & 3 & 0 \end{array} \quad (2x-3)(x-1) \rightarrow (2x-3)(x-1)(x+2)$$

$$x = 3/2, 1, -2$$

14. List all the possible rational zeros for the following functions:

a.  $2x^3 - x^2 - 7x + 6$

b.  $3x^4 - 5x^3 + 2x - 8$

$$\pm \frac{1, 2, 3, 6}{1, 2} = \pm 1, 2, 3, 6, 1/2, 3/2$$

15. A function of degree 3 has two zeros that are  $x = 4$  and  $x = -2$ . The third zero of the function must be...

- a. an imaginary zero  
c. no way to tell

**b. a real zero**

d. could be real or imaginary, depending on the function

16. Use the following information about the function  $f(x)$ .

$$f(0) = -3 \quad f(7) = -4 \quad f(3) = 0 \quad f(-1) = 4 \quad f(-4) = 0 \quad f(-2) = 0$$

a. what is the y-intercept? (0, -3)

b. what are the zeros of  $f(x)$ ?  $x = 3, -4, -2$

c. what are the factors of  $f(x)$ ?  $(x-3)(x+4)(x+2)$

d. what is the least possible degree of  $f(x)$ ? 3

e. if  $f(x)$  is divided by  $(x+1)$ , what will the remainder be?  $f(-1) = 4$

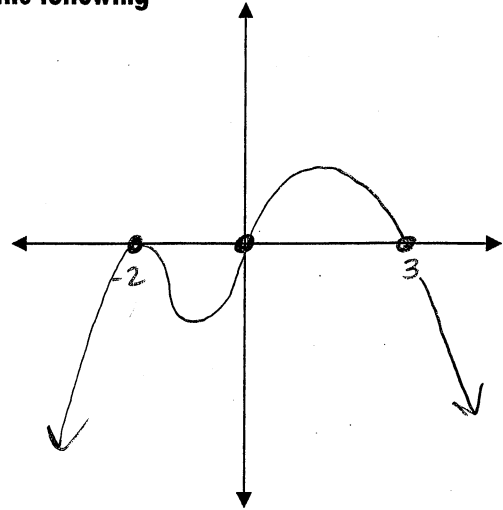
17. Sketch a graph of the following polynomial and answer the following

$$f(x) = -\frac{1}{5}x(x+2)(x-3)^2$$

Zeros	Multiplicity	Cross/Bounce
0	1	C
-2	1	C
3	2	B

Y-int:  $(0,0)$

- What is the degree of the function? 4
- Is the leading coefficient pos/neg? neg
- Describe the end behavior.  
 as  $x \rightarrow -\infty, f(x) \rightarrow -\infty$   
 as  $x \rightarrow \infty, f(x) \rightarrow -\infty$



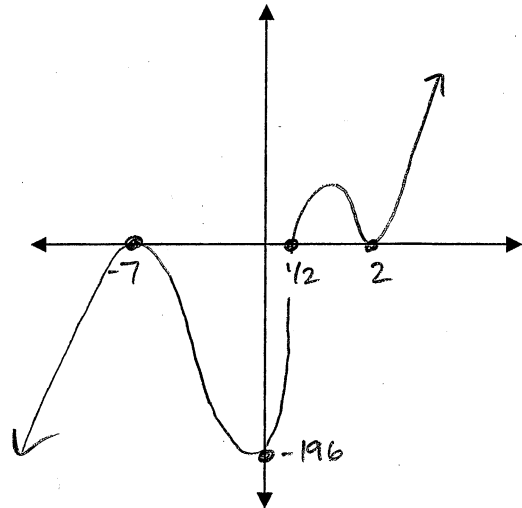
18. Sketch a graph of the following polynomial and answer the following

$$f(x) = (x-2)^2(x+7)^2(2x-1)$$

Zeros	Multiplicity	Cross/Bounce
2	2	B
-7	2	B
1/2	1	C

Y-int:  $(4)(49)(-1) = (0, -196)$

- What is the degree of the function? 5
- Is the leading coefficient pos/neg? pos
- Describe the end behavior.  
 as  $x \rightarrow -\infty, f(x) \rightarrow -\infty$   
 as  $x \rightarrow \infty, f(x) \rightarrow \infty$



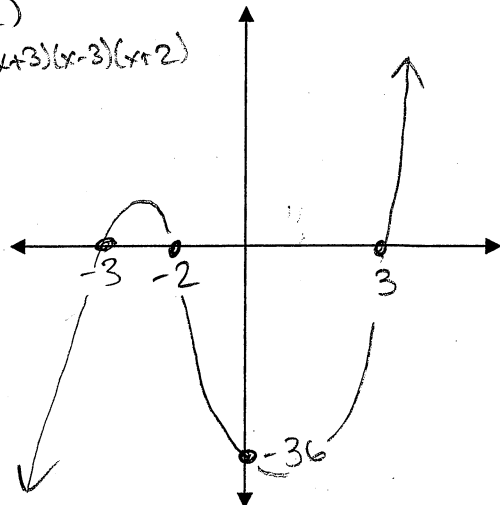
19. Sketch a graph of the following polynomial and answer the following

$$f(x) = 2x^3 + 4x^2 - 18x - 36 = 2x^2(x+2) - 18(x+2) = 2(x^2-9)(x+2) = 2(x+3)(x-3)(x+2)$$

Zeros	Multiplicity	Cross/Bounce
-3	1	C
3	1	C
-2	1	C

Y-int:  $(0, -36)$

- What is the degree of the function? 3
- Is the leading coefficient pos/neg? pos
- Describe the end behavior.  
 as  $x \rightarrow -\infty, f(x) \rightarrow -\infty$   
 as  $x \rightarrow \infty, f(x) \rightarrow \infty$

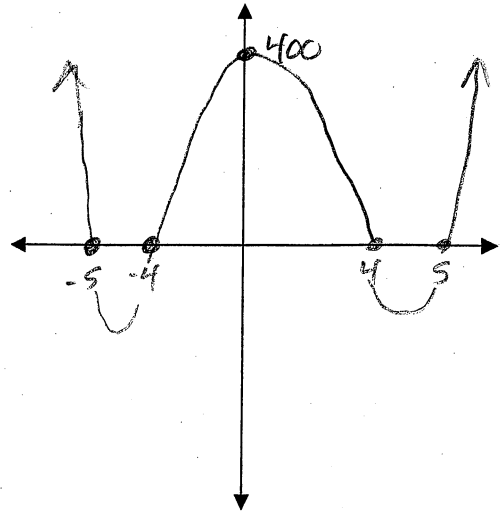


20. Sketch a graph of the following polynomial and answer the following

$$f(x) = x^4 - 41x^2 + 400$$

$$(x^2 - 16)(x^2 - 25) = (x+4)(x-4)(x+5)(x-5)$$

Zeros	Multiplicity	Cross/Bounce
-5	1	C
-4	1	C
4	1	C
5	1	C



Y-int:  $(0, 400)$

- What is the degree of the function? 4
- Is the leading coefficient pos/neg? pos
- Describe the end behavior  
 as  $x \rightarrow -\infty, f(x) \rightarrow \infty$   
 as  $x \rightarrow \infty, f(x) \rightarrow \infty$

21. Given the following graph, answer the following

- Name the zeros of the function and their multiplicity.

Zero: -9    -2    3    6  
 Multi: 1    2    1    1

- What is the least possible degree of the polynomial?

5

- Describe the end behavior.

as  $x \rightarrow -\infty, f(x) \rightarrow -\infty$   
 as  $x \rightarrow \infty, f(x) \rightarrow \infty$

- Local Minimums

$(-2, 0)$      $(5, -4)$

- Local Maximums

$(-6, 6)$      $(1, 2.5)$

- Absolute Minimum

$-\infty$

- Absolute Maximum

$\infty$

- Interval of Increase

$(-\infty, -6) \cup (-2, 1) \cup (5, \infty)$

- Interval of Decrease

$(-6, -2) \cup (1, 5)$

