

Solving Rational Inequalities WS 1

1.

$$\sqrt{0}: \frac{5}{0-2} = -\frac{5}{2} \quad \sqrt{3}: \frac{5}{3-2} = \frac{5}{1} = 5 \quad \text{pos}$$

neg ☹

$$(-\infty, 2)$$

2.

$$\frac{x-5}{x+3} - 1 > 0$$

$$\frac{x-5}{x+3} - \frac{x+3}{x+3} > 0$$

$$\frac{x-5-x-3}{x+3} > 0$$

$$\frac{-8}{x+3} > 0$$

$$\sqrt{-4}: \frac{-9}{-1} = 9 \quad \sqrt{4}: \frac{-8}{1} = -8 = -1 = \text{neg}$$

$$(-\infty, -3)$$

3.

$$\frac{x^2-3x+2}{x-3} - \frac{x(x-3)}{1(x-3)} < 0$$

$$\frac{\cancel{x^2}-3x+2-\cancel{x^2}+\cancel{3x}}{x-3} < 0$$

$$\frac{2}{x-3} < 0$$

$$\sqrt{0}: \frac{0-0+2}{0-3} = -\frac{2}{3} \quad \sqrt{4}: \frac{16-12+2}{4-3} = \frac{4+2}{1} = 6$$

$$-\frac{2}{3} = 0 = \text{neg} \quad \text{pos} \quad \text{pos}$$

$$6-4=2 \text{ pos}$$

$$(-\infty, 3)$$

4.

$$\sqrt{-3}: \frac{10}{-1} = -10 \quad \sqrt{3}: \frac{10}{5} = 2 \quad \text{pos}$$

$$-10 = \text{neg}$$

$$(-2, \infty)$$

5.

$$-2x-3=0$$

$$-2x=3$$

$$x = -\frac{3}{2}$$

$$\sqrt{-2}: \frac{4-3}{-6} = -\frac{1}{6} \quad \sqrt{2}: -\frac{3}{2}$$

$$-\frac{1}{6} = \text{neg}$$

$$\sqrt{0}: 4 \quad \sqrt{5}: \frac{-3}{-4} = \frac{3}{4} = \text{pos}$$

$$\sqrt{5}: \frac{-10-3}{1} = -13 = \text{neg}$$

$$(-\frac{3}{2}, 4)$$

$$6. \frac{x^2 - 4x + 8}{x-1} - \frac{x(x-1)}{x-1} < 0$$

$$\frac{x^2 - 4x + 8 - x^2 + x}{x-1} < 0$$

$$\frac{-3x + 8}{x-1} < 0$$

$$x = \frac{8}{3}, 1$$

$(-\infty, 1)$

∞: 0-0+8
0-1
-8-0=-8

neg

∞: 2-1
4-8+8
2-1

4-0=4
pos

∞: 9-12+8

2
2+3=pos

$$7. \frac{-4}{-1} = 4 \text{ pos} \quad | \quad \frac{-4}{5} \text{ neg} \quad | \quad (-5, \infty)$$

$$8. \frac{-4}{3} \text{ neg} \quad | \quad \frac{4}{1} = 4 \text{ pos} \quad | \quad (-\infty, 3)$$

$$9. \frac{8}{x^2+1} - \frac{4(x^2+1)}{x^2+1} \geq 0$$

$$\frac{8-4x^2-4}{x^2+1} \geq 0$$

$$\frac{-4x^2+4}{x^2+1} \geq 0$$

$$\frac{-4(x^2-1)}{x^2+1} \geq 0$$

∞: 8-4=4 pos
-1: 8-4=4 pos
∞: 8-4=4 pos
Check 2: 8/5 - 4 = neg

$(-1, 1)$

$$10. \frac{20}{x^2+1} - 2 \frac{(x^2+1)}{x^2+1} < 0$$

$$\frac{20}{x^2+1} - 2 \frac{(x^2+1)}{x^2+1} < 0$$

$$20 - 2x^2 - 2 < 0$$

$$x^2 + 1$$

$$-2x^2 + 18 < 0$$

$$x^2 + 1$$

$$-2(x^2 - 9) < 0$$

$$x^2 + 1$$

$$\sqrt{-4}:$$

$$\frac{20-2}{17}$$

neg :)

$$-3$$

$$\sqrt{0}:$$

$$\frac{20-2}{1}$$

pos :)

$$3$$

$$\sqrt{4}:$$

$$\frac{20-2}{17}$$

neg :)

$$(-\infty, -3) \cup (3, \infty)$$

$$11. \frac{3x+2}{x-1} + \frac{2(x-1)}{x-1} < 0$$

$$\frac{3x+2}{x-1} + \frac{2(x-1)}{x-1} < 0$$

$$3x+2+2x-2 < 0$$

$$x-1$$

$$5x < 0$$

$$x-1$$

$$\sqrt{-1}:$$

$$\frac{-1}{-2} + 2$$

pos :)

$$0$$

$$\sqrt{\frac{1}{3}}:$$

$$\frac{1+2}{-7/3}$$

pos :)

$$1$$

$$\sqrt{2}:$$

$$\frac{8}{1} + 2$$

pos :)

$$3 - \frac{3}{2} + 2$$

$$-\frac{9}{2} + 2$$

neg :)

$$(0, 1)$$

$$12. \frac{3x+2}{x-1} - \frac{x(x-1)}{x-1} > 0$$

$$\frac{3x+2}{x-1} - \frac{x(x-1)}{x-1} > 0$$

$$3x+2-x^2+x > 0$$

$$x-1$$

$$-x^2 + 4x + 2 > 0$$

$$x-1$$

$$-(x^2 - 4x - 2) > 0$$

$$x-1$$

$$\sqrt{-1}:$$

$$\frac{-1}{-2} + 1$$

pos :)

$$\sqrt{0}:$$

$$\frac{2}{-1} - 0$$

neg :)

$$1$$

$$\sqrt{2}:$$

$$\frac{8}{1} - 8$$

pos :)

$$\sqrt{5}:$$

$$\frac{17}{1} - 5(x)$$

neg :)

$$(-\infty, 2-\sqrt{6}) \cup (1, 2+\sqrt{6})$$

$$x^2 - 4x - 2$$

$$4 \cdot 1 = 2$$

$$-b \pm \sqrt{b^2 - 4ac} = 4 \pm \sqrt{16 + 8}$$

$$\frac{2a}{2}$$

$$\frac{2}{2}$$

$$= \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6} \approx 4.4$$

$$-0.4$$

13.

$$\frac{-1}{x+5} - \frac{2(x+5)}{x+5} \leq 0$$

$$\frac{-1 + 2x + 10}{x+5} \leq 0$$

$$\frac{2x+9}{x+5} \leq 0$$

$$(-2, \infty)$$

14.

$$\frac{-1}{x+5} - \frac{2(x+5)}{x+5} \leq 0$$

$$\frac{-1 + 2x + 10}{x+5} \leq 0$$

$$\frac{2x+9}{x+5} \leq 0$$

$$\begin{array}{c} | \\ \hline \sqrt{-6}: -5 \quad -4.75 \quad -9/2 \quad \sqrt{0}: \\ \frac{-1}{-1} + 2 \quad \frac{-1}{-0.25} + 2 \quad \frac{-1}{5} + 2 \\ \text{Pos} \quad \text{neg} \quad \text{Pos} \\ \text{":} \quad \text{":} \quad \text{":} \end{array}$$

$$(-5, -\frac{9}{2})$$

15.

$$\frac{2(x+3)}{x+2} - \frac{1(x+2)}{x+3} > 0$$

$$\frac{2x+6-x-2}{(x+2)(x+3)} > 0$$

$$\frac{x+4}{(x+2)(x+3)} > 0$$

$$\begin{array}{c} | \quad | \quad | \\ \hline \sqrt{-5}: -4 \quad \sqrt{-3.5}: -3 \quad \sqrt{-2.5}: -2 \quad \sqrt{0}: \\ \frac{2}{-3} + \frac{1}{12} \quad \frac{2}{-1.5} + \frac{1}{1.5} \quad \frac{2}{-1.5} - \frac{1}{1.5} \quad \frac{2}{2} - \frac{1}{3} \\ \text{neg} \quad \text{pos} \quad \text{pos} \quad \text{pos} \\ \text{":} \quad \text{":} \quad \text{":} \quad \text{":} \end{array}$$

$$(-4, \infty)$$

16.

$$\frac{5(x+4)}{x-4} - \frac{1(x-4)}{x+4} < 0$$

$$\frac{5x+20-x+4}{(x+4)(x-4)} < 0$$

$$\frac{4x+24}{(x+4)(x-4)} < 0$$

$$\frac{4(x+6)}{(x+4)(x-4)} < 0$$

$$\begin{array}{c} | \quad | \quad | \quad | \\ \hline \sqrt{-7}: -6 \quad \sqrt{-5}: -4 \quad \sqrt{0}: 4 \quad \sqrt{5} \\ \frac{5}{-11} + \frac{1}{13} \quad \frac{5}{-9} + \frac{1}{11} \quad \frac{5}{-4} - \frac{1}{4} \quad \frac{5}{1} - \frac{1}{4} \\ \text{neg} \quad \text{pos} \quad \text{neg} \quad \text{pos} \\ \text{":} \quad \text{":} \quad \text{":} \quad \text{":} \end{array}$$

$$(-\infty, -6) \cup (-4, 4)$$

$$17. \frac{5(x+2)}{x+3} - \frac{4(x+3)}{x+2} \geq 0$$

$$\frac{5x+10-4x-12}{(x+3)(x+2)} \geq 0$$

$$\frac{x+2}{(x+3)(x+2)} \geq 0$$

√-4:	-3	√-2.5:	-2	√0:
-2		-5		2
(-1)(-2)		(1.5)(-5)		(3)(2)
neg		pos ☹		pos ☺

$$(-3, \infty)$$

$$18. \frac{2(x^3)}{x+6} + \frac{3(x+6)}{x-3} > 0$$

$$\frac{2x-6+3x+18}{(x+6)(x-3)} > 0$$

$$\frac{5x+12}{(x+6)(x-3)} > 0$$

√-7:	-6	√-3:	-12/5	√0:	3	√4:
2/1 + 3/-10		2/3 + 3/-6		2/6 + 3/3		2/10 + 3/1
neg		pos ☺		neg		pos ☺

$$\boxed{(-6, -\frac{12}{5})(3, \infty)}$$