

Rational Relationships Card Swap – Solutions with Work

$$1) \frac{27x^3+8}{9x^2-4} = \frac{(3x+2)(9x^2-6x+4)}{(3x+2)(3x-2)} = \boxed{\frac{9x^2-6x+4}{3x-2}}$$

$$2) \frac{\frac{4}{x+3}-1}{\frac{x-1}{5}} = \frac{\frac{4-1(x+3)}{x+3}}{\frac{x-1}{5}} = \frac{\frac{4-x-3}{x+3}}{\frac{x-1}{5}} = \frac{\frac{1-x}{x+3}}{\frac{x-1}{5}} = \frac{-(x-1)}{(x+3)} \cdot \frac{5}{(x-1)} = \boxed{-\frac{5}{x+3}}$$

$$3) \frac{8x-3}{4x+5} = \frac{3}{5}$$

$$5(8x - 3) = 3(4x + 5)$$

$$40x - 15 = 12x + 15$$

$$28x = 30$$

$$\boxed{x = \frac{15}{14}}$$

$$4) \frac{4x^2-9}{6x^2+x-12} = \frac{(2x+3)(2x-3)}{(3x-4)(2x+3)}$$

$$\boxed{\begin{array}{l} a) x \neq \frac{4}{3} \text{ and } x \neq -\frac{3}{2} \\ b) \frac{(2x-3)}{(3x-4)} \end{array}}$$

$$5) \frac{4x^2-9}{6x^2+x-12} \div \frac{2x^2-11x+12}{3x^2-12x} = \frac{(2x+3)(2x-3) \cdot 3x(x-4)}{(3x-4)(2x+3)(2x-3)(x-4)} = \boxed{\frac{3x}{3x-4}}$$

$$6) \frac{5x}{4-2x} - \frac{x}{6x-12} = \frac{5x}{2(2-x)} - \frac{x}{6(x-2)} = \frac{-15x}{6(x-2)} - \frac{x}{6(x-2)} = \frac{-16x}{6(x-2)} = \boxed{-\frac{8x}{3(x-2)}}$$

$$7) \frac{2}{y+2} + \frac{y}{y-2} = \frac{y^2+4}{y^2-4}$$

$$\frac{2}{y+2} + \frac{y}{y-2} = \frac{y^2+4}{(y+2)(y-2)}$$

$$2(y-2) + y(y+2) = y^2 + 4$$

$$2y - 4 + y^2 + 2y = y^2 + 4$$

$$4y = 8 \quad \dots \quad y \neq 2$$

\emptyset extraneous solution!

$$8) \frac{5-x}{(2x-1)(x+1)} = \frac{A}{(2x-1)} + \frac{B}{(x+1)}$$

$$-x + 5 = A(x+1) + B(2x-1)$$

$$-x + 5 = Ax + A + 2Bx - B$$

$$\begin{cases} -1 = A + 2B \\ 5 = A - B \end{cases} \dots \begin{cases} -1 = A + 2B \\ 10 = 2A - 2B \end{cases}$$

$$9 = 3A \dots A = 3$$

$$-1 = 3 + 2B$$

$$-4 = 2B$$

$$B = -2$$

$$\frac{3}{(2x-1)} - \frac{2}{(x+1)}$$

$$9) \frac{x^3 - xy^2}{2x^3 + 5x^2 - 2xy^2 - 5y^2} = \frac{x(x^2 - y^2)}{x^2(2x+5) - y^2(2x+5)} = \frac{x(x^2 - y^2)}{(2x+5)(x^2 - y^2)} = \frac{x}{2x+5}$$

$$10) \frac{\frac{2x}{3} - 2}{1 - 3x^{-1}} = \frac{\frac{2x}{3} - 2}{1 - \frac{3}{x}} = \frac{\frac{2x-2(3)}{3}}{\frac{x-3}{x}} = \frac{\frac{2x-6}{3}}{\frac{x-3}{x}} = \frac{2(x-3)}{3} \cdot \frac{x}{(x-3)} = \frac{2x}{3}$$

$$11) \frac{x+4}{x} + \frac{3}{x-4} = \frac{-16}{x^2-4x}$$

$$\frac{x+4}{x} + \frac{3}{x-4} = \frac{-16}{x(x-4)}$$

$$(x+4)(x-4) + 3(x) = -16$$

$$x^2 - 16 + 3x = -16$$

$$x^2 + 3x = 0$$

$$x(x+3) = 0$$

$$x \neq 0 \quad \boxed{x = -3}$$

$$12) \frac{2x^3 - x^2 + x + 5}{x^2 + 3x + 2} = 2x - 7 + \frac{18x + 19}{x^2 + 3x + 2} = 2x - 7 + \frac{18x + 19}{(x+2)(x+1)}$$

$$\frac{18x+19}{(x+2)(x+1)} = \frac{A}{(x+2)} + \frac{B}{(x+1)}$$

$$18x + 19 = A(x+1) + B(x+2)$$

$$18x + 19 = Ax + A + Bx + 2B$$

$$\begin{cases} 18 = A + B \\ 19 = A + 2B \end{cases} \dots \begin{cases} -18 = -A - B \\ 19 = A + 2B \end{cases}$$

$$1 = B$$

$$18 = A + 1$$

$$17 = A$$

$$2x - 7 + \frac{17}{(x+2)} + \frac{1}{(x+1)}$$