

Rat'l Rel. Test Review

1)

2) $\frac{x^2+20x-45}{x^3-2x^2-15x} \rightarrow \frac{x(x^2-2x-15)}{x(x+3)(x-5)}$

$$\frac{x^2+20x-45}{x(x+3)(x-5)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-5}$$

$$x^2+20x-45 = A(x+3)(x-5) + Bx(x-5) + Cx(x+3)$$

$$x^2+20x-45 = A(x^2-2x-15) + Bx(x-5) + Cx(x+3)$$

$$x^2+20x-45 = Ax^2-2Ax-15A + Bx^2-5Bx + Cx^2+3Cx$$

$$\begin{cases} 1 = A+B+C & \text{--- ①} \\ 20 = -2A-5B+3C & \text{--- ②} \\ -45 = -15A & \end{cases} \rightarrow \begin{cases} 1 = 3+B+C & \text{--- ③} \\ 20 = -6-5B+3C & \text{--- ④} \\ -2 = B+C & \text{--- ⑤} \\ 26 = -5B+3C & \text{--- ⑥} \end{cases}$$

$$\begin{aligned} A &= 3 \quad \text{--- ①} \\ \text{--- ③} \quad -10 &= 5B+5C & \text{--- ④} \quad -2 &= B+2 \\ 26 &= -5B+3C & \quad \quad \quad -4 &= B \\ \hline 16 &= 8C \\ 2 &= C \end{aligned}$$

$$\boxed{\frac{3}{x} - \frac{4}{x+3} + \frac{2}{x-5}}$$

3) $\frac{3}{3(x-4)} + \frac{2x}{3(16-x^2)} = \frac{5}{x+4}$

$$3(x+4)(x+4) \left(\frac{3}{3(x-4)} + \frac{-2x}{3(x+4)(x-4)} = \frac{5}{(x+4)} \right)$$

$$\begin{aligned} 3(x+4) - 2x &= 5 \cdot 3(x-4) \\ 3x+12-2x &= 15x-60 \\ 72 &= 14x \\ \frac{72}{14} &= x \end{aligned}$$

$$\boxed{x = \frac{36}{7}}$$

$$4) \frac{\frac{x}{y^2} + \frac{y}{x^2}}{\frac{1}{x^2} - \frac{1}{y^2}} = \frac{\frac{x^3 + y^3}{x^2 y^2}}{\frac{y^2 - x^2}{x^2 y^2}}$$

$$= \frac{x^3 + y^3}{x^2 y^2} \cdot \frac{x^2 y^2}{y^2 - x^2} = \frac{(x+y)(x^2 - xy + y^2)}{(y-x)(y+x)}$$

$$= \boxed{\frac{x^2 - xy + y^2}{y-x}}$$

$$5) \textcircled{N} \frac{x^3 + x^2 y - xy^4 - y^5}{x^2(x+y) - y^4(x+y)} = \frac{(x+y)(x+y^2)(x-y^2)}{(x+y)(x+y^2)}$$

$$= \boxed{x-y^2}$$

$$\textcircled{D} \frac{x^2 + xy^2 + xy + y^3}{x(x+y^2) + y(x+y^2)} = \frac{(x+y^2)(x+y)}{(x+y^2)(x+y)}$$

$$6) \frac{10x(x-2)}{40x^2(x-2)} \cdot \frac{4x(6x^2+5)}{(6x^2+5)} = \frac{40x^2}{40x^2} = \boxed{1}$$

$$7) \frac{1}{3(x+1)} + \frac{1}{9x}$$

$$\frac{3x \cdot 1}{3x \cdot 3(x+1)} + \frac{1(x+1)}{9x(x+1)} = \frac{3x + x + 1}{9x(x+1)}$$

$$= \boxed{\frac{4x+1}{9x(x+1)}}$$

10)

$$\begin{array}{r}
 x^2 - 2x - 8 \overline{) 2x^3 - 4x^2 - 15x + 5} \\
 \underline{-(2x^3 - 4x^2 - 16x)} \\
 x + 5
 \end{array}$$

$$2x + \frac{x+5}{x^2-2x-8} = 2x + \frac{x+5}{(x-4)(x+2)}$$

$$\frac{x+5}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$$

$$x+5 = A(x+2) + B(x-4)$$

$$x+5 = Ax + 2A + Bx - 4B$$

$$\begin{cases}
 1 = A + B \\
 5 = 2A - 4B
 \end{cases}
 \rightarrow
 \begin{cases}
 4 = 4A + 4B \\
 5 = 2A - 4B
 \end{cases}$$

$$9 = 6A$$

$$\frac{3}{2} = A$$

$$1 = \frac{3}{2} + B$$

$$\frac{2}{2} - \frac{3}{2} = B$$

$$-\frac{1}{2} = B$$

$$\boxed{2x + \frac{3}{2(x-4)} - \frac{1}{2(x+2)}}$$

$$\begin{aligned}
 8) \quad & \frac{x^2}{x+3} - \frac{3x}{-(x-3)} \\
 & = \frac{x^2(x-3)}{(x+3)(x-3)} + \frac{3x(x+3)}{(x-3)(x+3)} \\
 & = \frac{x^3 - 3x^2 + 3x^2 + 9x}{(x+3)(x-3)} = \frac{x^3 + 9x}{(x+3)(x-3)} \\
 & = \boxed{\frac{x(x^2+9)}{(x+3)(x-3)}}
 \end{aligned}$$

$$9) (x+2) \left[\frac{x}{x+2} + \frac{x}{1} = \frac{5x+8}{x+2} \right]$$

$$x + x(x+2) = 5x+8$$

$$x + x^2 + 2x = 5x + 8$$

$$x^2 + 3x = 5x + 8$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$\boxed{x=4} \quad \cancel{x=-2}$$

10) next page!