

Name:

Period:

Date:

Practice Worksheet: Rational Functions

- The graph of a rational function has a(n) horizontal asymptote at  $y = 0$  if the degree of the numerator is less than the degree of the denominator.
- The horizontal asymptote of a rational function is the ratio of leading coefficients when the degree of the numerator is equals the degree of the denominator.
- If the degree of the numerator is more than the degree of the denominator, the rational function has a(n) horizontal or oblique asymptote. (can't have both)
- You must use synthetic or long division to find the equation of an oblique asymptote.
- When you cancel a common factor out of the numerator and denominator of a rational function, it forms a hole in the graph at that point. To find the coordinates of that point, set the canceled factor equal to zero and solve for x. Then substitute result back into the simplified form to find y.

6] Match the work shown for each process.

- j(x) a) Finding an oblique asymptote      g(x) b) Finding a zero  
h(x) c) Finding a vertical asymptote      f(x) d) Finding a hole

$f(x) = \frac{x^2 - 8x + 7}{(x-1)(x-7)}$ $\frac{x^2 - 8x + 7}{x-1} = x-7$ $x-1=0 \quad y=1-7=-6$ $x=1$	$g(x) = \frac{x^2 - 15x + 56}{x-3}$ $g(x) = \frac{(x-7)(x-8)}{x-3}$ $x-7=0 \quad x-8=0$ $x=7 \quad x=8$
$h(x) = \frac{x^2 + 2x - 15}{2x^2 - 7x + 3}$ $\frac{(x+5)(x-3)}{(2x-1)(x-3)}$ $2x-1=0$ $x=1/2$	$j(x) = \frac{2x^2 - 5x + 5}{x-2}$ $2 \overline{) 2 \ -5 \ 5}$ $2 \ -1 \ 3 \ R$ $y=2x-1$

7]  $f(x) = \frac{x^2 - 25}{x^2 - 4x - 5} = \frac{(x+5)(x-5)}{(x-5)(x+1)} = \frac{x+5}{x+1}$

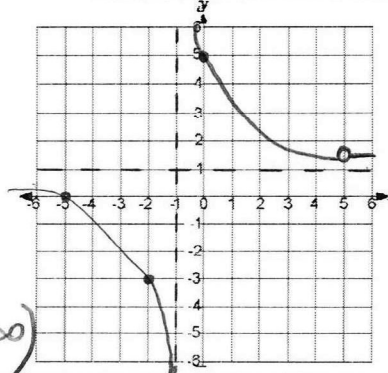
Coordinates of hole:  $(5, \frac{5}{3})$

Equation of vertical asymptote:  $x = -1$

Zero(s):  $(-5, 0)$       y-intercept:  $(0, 5)$

Equation of horizontal OR oblique asymptote:  $y = 1$

Domain:  $(-\infty, -1) \cup (-1, 5) \cup (5, \infty)$       Range:  $(-\infty, 1) \cup (1, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$



8]  $f(x) = \frac{x^2 + x - 6}{x^2 - 4} = \frac{(x+3)(x-2)}{(x+2)(x-2)} = \frac{x+3}{x+2}$

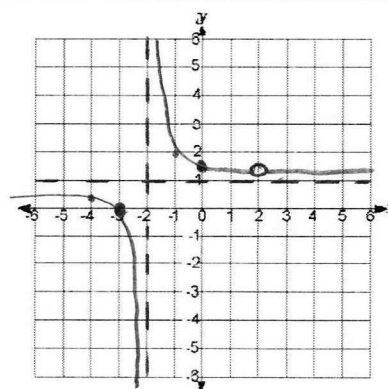
Coordinates of hole:  $(2, \frac{5}{4})$

Equation of vertical asymptote:  $x = -2$

Zero(s):  $(-3, 0)$       y-intercept:  $(0, \frac{3}{2})$

Equation of horizontal OR oblique asymptote:  $y = 1$

Domain:  $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$       Range:  $(-\infty, 1) \cup (1, \frac{5}{4}) \cup (\frac{5}{4}, \infty)$



\* 9]  $f(x) = \frac{x^2+2x-3}{x-1} = \frac{(x+3)(x-1)}{x-1} = \frac{x+3}{1}$

Coordinates of hole: (1, 4)

Equation of vertical asymptote: none

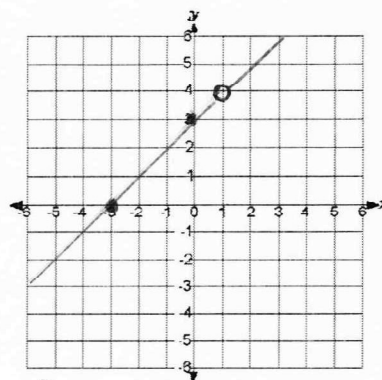
Zero(s): (-3, 0) y-intercept: (0, 3)

Equation of horizontal OR oblique asymptote:  $y = x + 3$

Domain:

Range:

$(-\infty, 1) \cup (1, \infty)$   $(-\infty, 4) \cup (4, \infty)$



10]  $f(x) = \frac{x-3}{x^2+x-12} = \frac{x-3}{(x+4)(x-3)} = \frac{1}{x+4}$

Coordinates of hole: (3, 1/7)

Equation of vertical asymptote:  $x = -4$

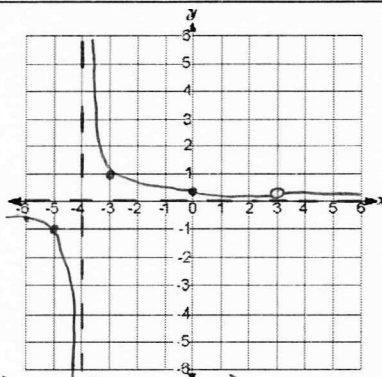
Zero(s): none y-intercept: (0, 1/4)

Equation of horizontal OR oblique asymptote:  $y = 0$

Domain:

Range:

$(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$   $(-\infty, 0) \cup (0, 1/7) \cup (1/7, \infty)$



11]  $f(x) = \frac{x^2+2x-15}{x+3} = \frac{(x+5)(x-3)}{(x+3)}$

Coordinates of hole: none

Equation of vertical asymptote:  $x = -3$

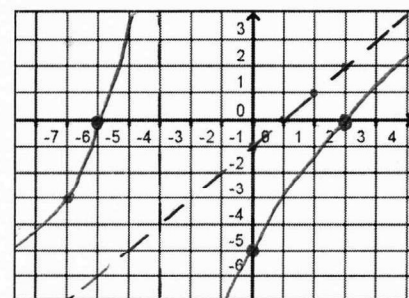
Zero(s): (-5, 0) (3, 0) y-intercept: (0, -5)

Equation of horizontal OR oblique asymptote:  $y = x - 1$

Domain:

Range:

$(-\infty, -3) \cup (-3, \infty)$   $(-\infty, \infty)$



12]  $f(x) = \frac{(x+1)(x^2+2x+1)}{x^2-1} = \frac{(x+1)(x+1)(x+1)}{(x+1)(x-1)} = \frac{(x+1)(x+1)}{x-1}$

Coordinates of hole: (-1, 0) graph will BOUNCE at the hole (-1, 0)

Equation of vertical asymptote:  $x = 1$

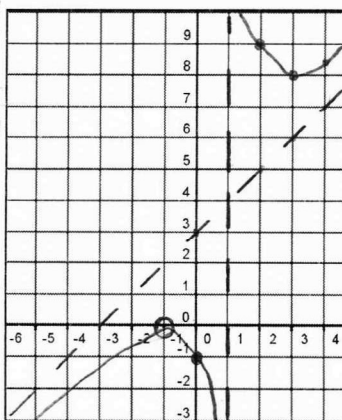
Zero(s): (-1, 0) multiplicity of 2 y-intercept: (0, -1)

Equation of horizontal OR oblique asymptote:  $y = x + 3$

Domain:

Range:

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$   $(-\infty, 0) \cup [8, \infty)$



Hint: The point (3, 8) is on the graph and it is a rel. minimum.

USE synthetic division

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