Properties of Logarithms

Special Logarithm Values

Let b be positive numbers such that $b \neq 1$.

$$\log_b 1 = 0$$

$$\log_b b = 1$$

Inverse Properties

Let b be positive numbers such that $b \neq 1$.

$$\log_b b^n = n$$

$$b^{\log_b n} = n$$

Properties of Logarithms

Let b, u, and v be positive numbers such that $b \neq 1$.

		Examples:
Product Property	$\log_b uv = \log_b u + \log_b v$	$\log_2 5x = \log_2 5 + \log_2 x$
Quotient Property	$\log_b \frac{u}{v} = \log_b u - \log_b v$	$\log_2\left(\frac{x}{5}\right) = \log_2 x - \log_2 5$
Power Property	$\log_b u^n = n \log_b u$	$\log_2 x^5 = 5 \cdot \log_2 x$

expand $\longrightarrow$

$\longleftarrow$ condense

Example 1:

Use $\log_a 5 \approx 0.732$ and $\log_a 11 \approx 1.091$ to approximate the following.

Substitute

a. $\log_a \frac{5}{11} = \log_a 5 - \log_a 11 = .732 - 1.091 = -0.359$

b. $\log_a 55 = \log_a(5 \cdot 11) = \log_a 5 + \log_a 11 = .732 + 1.091 = 1.823$

c. $\log_a 25 = \log_a(5^2) = 2 \cdot \log_a 5 = 2(.732) = 1.464$

Example 2:
Expand each logarithm. Assume x is positive.

a. $\log_2 \frac{x}{2}$

Quotient

$= \log_2 x - \log_2 2$

$= \log_2 x - 1$

b. $\log_3 9x$

Product

$\log_3 9 + \log_3 x$

Power

$\log_3 3^2 + \log_3 x$

$2 \cdot \log_3 3 + \log_3 x$

$2 + \log_3 x$

c. $\log_5 2x^6$

Product

$\log_5 2 + \log_5 x^6$

Power

$\log_5 2 + 6 \cdot \log_5 x$

Example 3:
Condense each logarithm. (write as a single log)

a. $\log_4 2 + \log_4 8$

Product

$\log_4 (2 \cdot 8)$

$\log_4 16$

b. $2\log_3 5 + \frac{1}{2}\log_3 x$

Power

$\log_3 5^2 + \log_3 x^{1/2}$

Product

$\log_3 (25\sqrt{x})$

c. $2\log_3 7 - 5\log_3 x$

Power

$\log_3 7^2 - \log_3 x^5$

Quotient

$\log_3 \left(\frac{49}{x^5}\right)$